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Finite-Difference Time-Domain Simulation of Light Propagation in 2D Periodic and Quasi-Periodic Photonic Structures

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Abstract

Ultra-short pulse is a promising technology for achieving ultra-high data rate transmission which is required to follow the increased demand of data transport over an optical communication system. Therefore, the propagation of such type of pulses and the effects that it may suffer during its transmission through an optical waveguide has received a great deal of attention in the recent years. We provide an overview of recent theoretical developments in a numerical modeling of Maxwell's equations to analyze the propagation of short laser pulses in photonic structures. The process of short light pulse propagation through 2D periodic and quasi-periodic photonic structures is simulated based on Finite-Difference Time-Domain calculations of Maxwell's equations.

1. Introduction

The finite-difference time-domain method (FDTD) is a very powerful technique for numerical analysis of Maxwell's equations [1]. Due to its accuracy, the FDTD method is widely used for simulation of light propagation in optical waveguides, scattering media or photonic crystals [2, 3].

Although the first studies of optical coupling processes in waveguide arrays were performed in early 1960's [4, 5], it was only recently that high-contrast dielectric elements became available which made possible creation of photonic crystal structures [6]. Recently, discretized light propagation in photonic lattices has attracted a lot of interest [7-11].

In this work, we have used the FDTD method to analyze the processes of light propagation in dielectric media with different microstructures embedded in them. In particular, the tasks of the light pulses propagation in two-dimensional (2D) coupled-waveguides, periodic and quasi-periodic structures have been examined numerically. The cases of different polarization state of electromagnetic waves (TE-, and TM-modes) have been studied, and the space distributions of the amplitudes for electric and magnetic vectors, as well as for the energy density of the electromagnetic field have been calculated. The influence of the parameters of the light pulses (wavelength, duration of pulse, transverse size of the beam) and the parameters of the medium (the values of refractive index of dielectric layers, its geometrical size and mutual arrangement) on transmission characteristics of the electromagnetic field have been analyzed. As a result, the main features of energy transition modes between two coupled waveguides ("pendulum mode"), and "discrete diffraction" in the system of several coupled waveguides have been studied thoroughly. In addition, the features of the short light pulses propagation in quasi-periodic photonic structures with a labyrinthine distribution of the refractive index have been analyzed based on the previously developed technique [12].

2. Theoretical model Light propagation: Finite-difference time-domain (FDTD) method

The finite-difference time-domain technique or FDTD-method is a widely used technique that numerically solves Maxwell's equations with a high accuracy, entailing a considerable computing time [1]. This method allows tracking the spatial periodic evolution of the electromagnetic field inside of the environment with an arbitrary distribution of the dielectric conductivity. In this section, we show that with the using of FDTD method we can analyze the propagation of short laser pulses in periodic and quasi-periodic structures.

Let's consider Maxwell's equations for an optical medium

$$-\mu_0 \frac{\partial \vec{H}}{\partial t} = \nabla \times \vec{E}\ , \tag{1}$$

$$\varepsilon_0 \varepsilon(x,y,z) \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{H}\ , \tag{2}$$

These equations can be decomposed into the three coordinate components to obtain a set of six differential equations:

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right], \tag{3}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right], \tag{4}$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu_0}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right], \tag{5}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon(x,y,z)}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right], \tag{6}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon(x,y,z)}\left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right], \tag{7}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon(x,y,z)}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]. \tag{8}$$

From now on, a 2D configuration will be considered, in such a way that the electrical permittivity is reduced to $\varepsilon = \varepsilon\,(x, y)$. With this situation, the differential equations for TE mode are:

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_z}{\partial y}\ , \tag{9}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial x}\ , \tag{10}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon(x,y)}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]. \tag{11}$$

The finite difference approximation of equation sets (9) – (11) implies a set of equations that can be solved explicitly [2]. According to FDTD theory, these differential functions are segregated in both space and time, so that they can be calculated as:

$$\begin{aligned} &H_x^{n+1/2}(i, j+1/2) = \\ &= H_x^{n-1/2}(i, j+1/2) - \frac{\Delta t}{\mu_0 \Delta y}\left\{E_z^n(i, j+1) - E_z^n(i, j)\right\}, \end{aligned} \tag{12}$$

$$H_y^{n+1/2}(i+1/2,j)= = H_y^{n-1/2}(i+1/2,j)-\frac{\Delta t}{\mu_0\Delta x}\left\{E_z^n(i+1,j)-E_z^n(i,j)\right\}, \quad (13)$$

$$E_z^{n+1}(i,j)=E_z^n(i,j)+ +\frac{\Delta t}{\varepsilon_0\varepsilon(i,j)}\left\{\frac{1}{\Delta x}\left[H_y^{n+1/2}(i+1/2,j)-H_y^{n+1/2}(i-1/2,j)\right]\right\}- \quad (14)$$
$$-\frac{\Delta t}{\varepsilon_0\varepsilon(i,j)}\left\{\frac{1}{\Delta y}\left[H_x^{n+1/2}(i,j+1/2)-H_x^{n+1/2}(i,j-1/2)\right]\right\}$$

In these equations Δx and Δy are the spatial mesh steps along the coordinates *x* and y, respectively, Δt is the time step. The following separation for the desired functions is used here:

$$F^n(i,j)=F(i\Delta x, j\Delta y, n\Delta t)=F(x,y,t).$$

On the other hand, the equations for TM mode are:

$$\frac{\partial E_x}{\partial t}=\frac{1}{\varepsilon_0\varepsilon(x,y)}\frac{\partial H_z}{\partial y}, \quad (15)$$

$$\frac{\partial E_y}{\partial t}=-\frac{1}{\varepsilon_0\varepsilon(x,y)}\frac{\partial H_z}{\partial x}, \quad (16)$$

$$\frac{\partial H_z}{\partial t}=-\frac{1}{\mu_0}\left[\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right]. \quad (17)$$

Again, after separation of these functions according to FDTD algorithm, we have:

$$E_x^{n+1}(i+1/2,j)=E_x^n(i+1/2,j)+ +\frac{\Delta t}{\varepsilon_0\varepsilon(i,j)}\frac{1}{\Delta y}\left\{\begin{matrix}H_z^{n+1/2}(i+1/2,j+1/2)-\\ -H_z^{n+1/2}(i+1/2,j-1/2)\end{matrix}\right\}, \quad (18)$$

$$E_y^{n+1}(i,j+1/2)=E_y^n(i,j+1/2)- -\frac{\Delta t}{\varepsilon_0\varepsilon(i,j)}\frac{1}{\Delta x}\left\{\begin{matrix}H_z^{n+1/2}(i+1/2,j+1/2)-\\ -H_z^{n+1/2}(i-1/2,j+1/2)\end{matrix}\right\}, \quad (19)$$

$$H_z^{n+1/2}(i+1/2,j+1/2)=H_z^{n-1/2}(i+1/2,j+1/2)- -\frac{\Delta t}{\mu_0}\left\{\begin{matrix}\frac{1}{\Delta x}\left[E_y^n(i+1,j+1/2)-E_y^n(i,j+1/2)\right]-\\ -\frac{1}{\Delta y}\left[E_x^n(i+1/2,j+1)-E_x^n(i+1/2,j)\right]\end{matrix}\right\} \quad (20).$$

3. Results and discussion

Numerical modeling of the problems for light beam propagation in the media with microstructures of refractive index has been performed for the following cases: 1) propagation of light pulses in the system of two coupled waveguides, 2) propagation of radiation in the system of N coupled waveguides, and 3) propagation of radiation in photonic structures with labyrinthine distribution of refractive index. In the numeric modeling, it is assumed that the source of the electromagnetic field in the form of a quasi-monochromatic wave with time pulse and spatial profile of Gaussian-like shape is started from the left border of the computation domain. The time evolution of spatial distribution of components of electric and magnetic fields, as well as the energy density has been calculated, and the results are presented in Figs. 1 – 3.

The mode of energy coupling of two closely positioned planar waveguides is demonstrated in Fig. 1. This task has been considered for the case, when the wavelength of incident electromagnetic wave $\lambda = 1\mu m$, pulse duration $\tau_p = 20\ fs$, diameter of waveguides $d_1 = d_2 = 1\mu m$, the distance between waveguides $d_0 = 1\mu m$. Planar waveguides are formed by the medium with refractive index n_2=1.5 (Fig. 1, a, dark area) embedded into the medium with refractive index n_1=1.45 (white area). The TE-polarization of radiation has been considered. Aswe can see, initially the light beam is focused to the bottom waveguide, and propagates along it (Fig. 1, b). After passing so-called coupling distance, the energy of the light pulse is almost completely transferred to the upper waveguide (Fig. 1, c) due to spatial intersection of transverse waveguide modes. The reverse process is taking place in doubled coupling distance (Fig. 1, d) providing the realization of "pendulum mode" for energy exchange between the two waveguides.

The typical results of numerical modeling for the mode of "discrete diffraction" in a system of N coupled waveguides are shown in Fig. 2 for TE-polarization of incident optical radiation. The wavelength of incident electromagnetic wave $\lambda = 1\mu m$, pulse duration $\tau_p = 20\ fs$. The system of

Planar waveguides (Fig. 2, a) are formed by the mediums with refractive index n_1=1.45 and n_2=1.5. The width of each waveguides $d_1 = 1\mu m$, and the distance between waveguides are $d_0 = 1\mu m$. Initially localized in the central waveguide (Fig. 2, b) the light pulse then is divided into two spatially symmetrical pulses localized in neighbor waveguides (Fig. 2, c). Next division of pulses

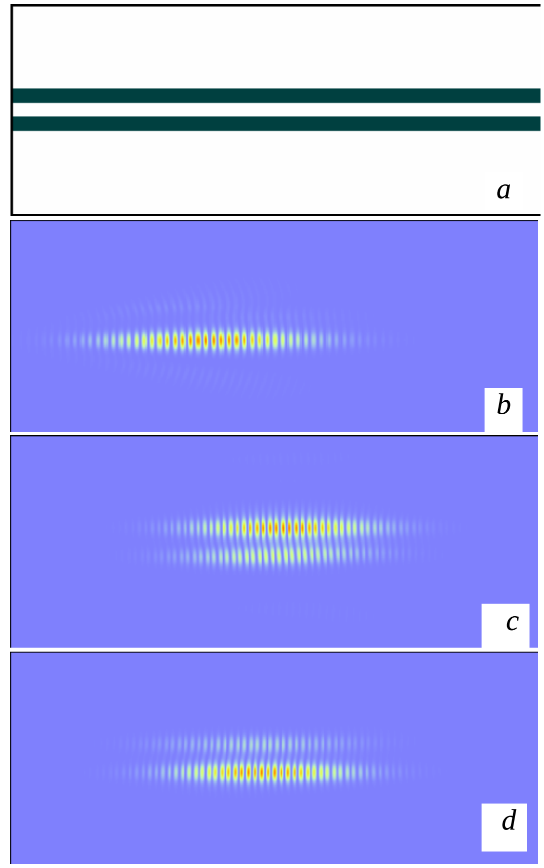

Fig. 1. Two coupled waveguides structure (*a*), and spatial distribution of the energy density of the electromagnetic field (*b* – *d*). t = 126 *fs* (*b*), 420 *fs* (*c*), 1000 *fs* (*d*). $\lambda = 1\mu m$, $\tau_p = 20\ fs$, $n_1 = 1.45$,$n_2 = 1.5$.

(Fig. 2, d) leads to energy transfer to the outer waveguides, while the coherent elimination of light pulse in the central waveguide takes place.

The analysis results of the quasi-monochromatic light pulses propagation kinetics in labyrinthine like photonic structures are presented in Fig. 3. The labyrinthine structures (Fig. 3, a) are characterized by the presence of short-distance and the absenceof long-distance order. Therefore, these structures can be considered as transient structures between photonic crystals, complex-structured waveguides,

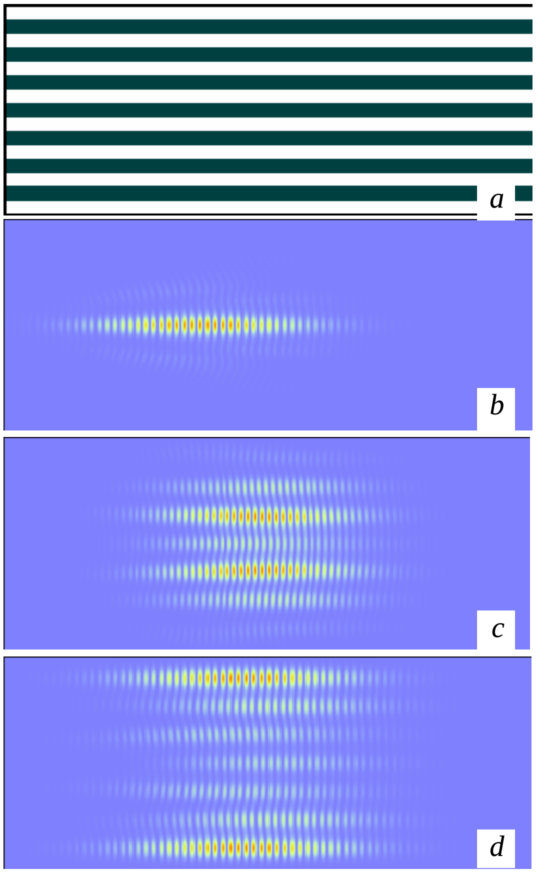

Fig. 2. coupled waveguides structure (*a*), and spatial distribution of the energy density of the electromagnetic field (*b* – *d*). t = 126 *fs* (*b*), 420 *fs* (*c*), 1000 *fs* (*d*). λ = 1μm, τ_p =20 *fs*, n_1 = 1.45,n_2 = 1.5.

and disordered systems. Based on the numerical experiments performed, it is shown that thedistribution of the energy density of the electromagnetic field is characterized by a complex branching structure (Fig. 3, b – e). The structures' application possibilities are analyzed with reference to increase the optical information storage time and localization of the electromagnetic field energy in the photonic structure [12].

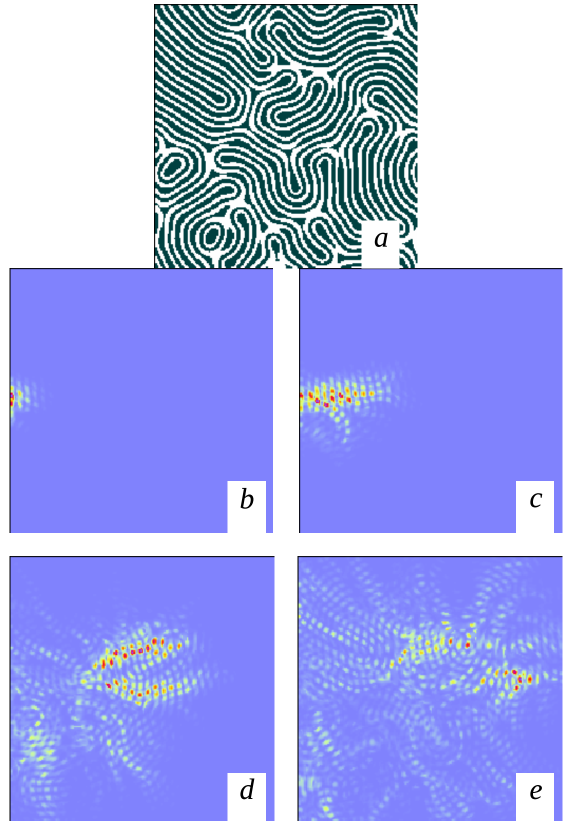

Fig. 3. Labyrinthine photonic structure (*a*), and spatial distribution of the energy density of the electromagnetic field. (*b* – *e*). t = 25 *fs* (b), 50 *fs* (c), 75 *fs* (*d*), 100 *fs* (*e*). λ = 1μm, τ_p =10 *fs*, n_1 = 1, n_2 = 2; the computational domain 10 λ – 10 λ.

4. Conclusion

This article provides an overview of theoretical analysis that has been performed in order to evaluate the light propagation in 2D photonic structures. We show that the FDTD method is well suited for studying of the pulse propagation in 2D photonic structures, and a very useful tool for investigating nonlinear pulse propagation and its interaction with the media.

References

[1] A. Taflove, S.C. Hagness, "Computational Electrodynamics: the Finite-Difference Time-Domain Method", 2000, Norwood, MA: Artech.

[2] K. Kawano, T. Kitoh, "Introduction to Optical Waveguide Analysis: Solving Maxwell's Equations

and the Schrödinger Equation”, 2001, John Willey and Sons, Inc.

[3] J.D. Joannopoulos, S.G. Johnson, J.N. Winn, R.D. Meade, “Photonic Crystals: Molding the Flow of Light”, 2008, Princeton University Press.

[4] A.L. Jones, J. Opt. Soc. Am. 55 (3) (1965) 261.

[5] S. Somekh, E. Garmire, A. Yariv, H.L. Garvin, R.G. Hunsperger, Appl. Phys Lett. 22 (1) (1973) 46–47.

[6] J.D. Joannopoulos, P.R. Villeneuve, S.H. Fan, Nature 386 (6621) (1997) 143–149.

[7] Y.V.Kartashov, B.A. Malomed, L. Torner, Rev. Modern Phys. 83 (1) (2011) 247–305.

[8] Y.V. Kartashov, V.A. Vysloukh, L. Torner, in: E. Wolf (Ed.), Progress in Optics, vol. 52, Elsevier,2009, pp. 63–148.

[9] Y.V. Kartashov, V.A. Vysloukh, L. Torner, Eur. Phys. J.-Spec. Top. 173 (2009) 87–105.

[10] S. Longhi, Laser Photonics Rev. 3 (3) (2009) 243–261.

[11] A. Szameit, S. Nolte, Journal of Physics B: Atomic, Molecular and Optical Physics 43 (16) (2010) 163001. URL: http://stacks.iop.org/0953-075/43/i=16/a=163001.

[12] O.G. Romanov, Bulletin of the Russian Academy of Sciences: Physics, 73 (2009) 1567.