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# Bending and Free Vibration Analysis of Nonlocal Functionally Graded Nanocomposite Timoshenko Beam Model Rreinforced by SWBNNT Based on Modified Coupled Stress Theory

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## Abstract

In this article, the bending and free vibration analysis of functionally graded (FG) nanocomposites Timoshenko beam model reinforced by single-walled boron nitride nanotube (SWBNNT) using micro-mechanical approach embedded in an elastic medium is studied. The modified coupled stress (MCST) and nonlocal elasticity theories are developed to take into account the size-dependent effect. The mechanical properties of FG boron nitride nanotube-reinforced composites are assumed to be graded in the thickness direction and estimated through the micro-mechanical approach. The governing equations of motion are obtained using Hamilton's principle based on Timoshenko beam theory. The Navier's type solution is implemented to solve the equations that satisfy the simply supported boundary conditions. Furthermore, the influences of the slenderness ratio, length of nanocomposite beam, material length scale parameter, nonlocal parameter, power law index, axial wave number, and Winkler and Pasternak coefficients on the natural frequency of nanocomposite beam are investigated. Also, the effect of material length scale parameter on the dimensionless deflection of FG nanocomposite beam is studied.



## 1. Introduction

Boron nitride nanotubes (BNNTs) due to the exceptional mechanical, electrical, and thermal properties are considered for the nanostructures. Chen et al. [1] presented a model of composite laminated Reddy beam based on modified coupled stress theory (MCST). They depicted the effects of micro-structures in their model and showed the obtained stress from Reddy beam model is lower than that of from Timoshenko and Euler - Bernoulli beam theories. Mosallaie et al. [2] analyzed the torsional buckling of the electro-thermo-mechanical cylindrical shells reinforced by polymer piezoelectric double-walled boron nitride nanotubes (DWBNNTs). They expressed properties of the electro-thermo-mechanical smart composite materials using a micro-mechanical approach. Their results stated that the critical buckling load increases with increasing the foam core. According to Mori–Tanaka approach, Ansari et al. [3] illustrated the free vibration analysis of micro-beams made of functionally graded materials (FGMs) based on the strain gradient Timoshenko beam theory. They assumed the material properties of the functionally graded (FG) beams to be graded in the thickness direction. Using Hamilton's principle, they derived the equations of motion together with corresponding boundary conditions for the free vibration analysis of FGM micro-beam. Moreover, they compared different beam models based on the classical theory (CT), MCST, and strain gradient theory (SGT) for various values of gradient index. It was shown from their results that the value of gradient index plays an important role in the vibrational response of the micro-beam for lower slenderness ratios. Simsek et al. [4] studied a micro scale FG Timoshenko beam model for static analysis based on MCST. They assumed the material properties of the FG micro-beam to be graded in the thickness direction and estimated through the Mori–Tanaka homogenization technique and the rule of mixture. Their results showed that the deflections of the micro-beam by the classical beam theory are always larger than those by MCST. Shariyat [5] investigated the free vibration and buckling analysis of FG rectangular plates subjected to the electro-thermo-mechanical loadings with surface-bonded or embedded piezoelectric sensors and actuators. His results showed that generally, initial geometric imperfections lead to an increase in the fundamental bending natural frequency and decreases the critical buckling load. Using Timoshenko beam model and von Kármán geometric nonlinearity, Ke et al. [6] developed the nonlinear free vibration of FG nanocomposite beams reinforced by the single-walled carbon nanotube (SWCNT). They considered the material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) to be graded in the thickness direction and estimated though the rule of mixture. Also, they applied the Ritz method to obtain the governing equation of motion which is then solved by a direct iterative method to derive the nonlinear vibration frequencies of FG-CNTRC beams with various boundary conditions. Their numerical results depicted the influences of nanotube volume fraction, vibration amplitude, slenderness ratio, end supports and carbon nanotube (CNT) distribution on the nonlinear free vibration characteristics of FG-CNTRC beams. Xiang et al. [7] studied the free and forced vibration analysis of FG laminated beam with variable thickness under thermal loading and initial stress based on Timoshenko beam theory. They considered the effects of the thickness variation, temperature change, slenderness ratio, volume fraction index, the thickness of FG layer, and boundary conditions on the natural frequencies. Simsek [8] investigated the analytical and numerical procedures for the free vibration of an embedded micro-beam under action of a moving micro-particle based on MCST within the framework of Euler–Bernoulli beam theory. He studied the influences of the material length scale parameter, Poisson's ratio, velocity of micro-particle and elastic medium constant on the natural frequency of the micro-beam. Rahmati and Mohammadimehr [9] analyzed the electro-thermo-mechanical vibration analysis of non-uniform and non-homogeneous boron nitride nanorod embedded in an elastic medium. They obtained the steady state heat transfer equation without external heat source for non-homogeneous rod. They investigated the effects of attached mass, lower and higher vibrational mode, elastic medium, piezoelectric coefficient, dielectric coefficient, cross section coefficient, non-homogeneity parameter and small-scale parameter on the natural frequency. Eltaher et al. [10] illustrated the size dependent effect on the static and buckling behavior of FG nano-beam based on the nonlocal elasticity theory. They used from the finite element method to discretize this model and obtain a numerical approximation for equilibrium equations. Ghorbanpour Arani et al. [11] studied the dynamic stability of SWCNT and double-walled carbon nanotube (DWCNT) under dynamic axial loading using the nonlocal elasticity theory and minimum principle of total potential energy. They obtained the critical dynamic axial load using the Rayleigh-Ritz method. Akgöz and Civalek [12] investigated the vibration response of non-homogeneous and non-uniform micro-beam in conjunction with Euler-Bernoulli beam and MCST. They used Rayleigh-Ritz method to obtain an approximate solution for the free transverse vibration. It is observed from their results that the influences of material properties on the natural frequencies of axially FG tapered microbeams are not negligible. Yas and Heshmati [13] presented the dynamic analysis of FG nanocomposite beams reinforced by randomly oriented straight SWCNTs under the actions of moving load. They used Timoshenko and Euler–Bernoulli beam theories to evaluate dynamic characteristics of the beam. Also, they employed the Eshelby–Mori–Tanaka approach based on an equivalent fiber to investigate the material properties of the nanocomposite beam. Their numerical results showed that the effects of various material distributions, CNT orientations, velocity of the moving load, shear deformation, slenderness ratios and boundary conditions on the dynamic characteristics of the nanocomposite beam.

In the present work, using Timoshenko beam model, the bending and free vibration analysis of the FG nanocomposite beam under electro-thermo-mechanical loadings based on the MCST is studied. The effects of the slenderness ratio, length of nanocomposite beam, material length scale parameter, nonlocal parameter, power law index, axial wave number, and Winkler and Pasternak coefficients on the natural frequency of FG nanocomposite beam reinforced by single-walled boron nitride nanotube (SWBNNT) are taken into account.

## 2. The constitutive equations of FG nanocomposite beam

The constitutive equations of electro-thermo-mechanical beam model based on nonlocal piezo-elasticity theory can be expressed as follows [14]:

$$\left(1-(e_0a)^2\nabla^2\right)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - h_{kij}E_k$$
$$D_m = h_{mkl}\varepsilon_{kl} + \in_{mk} E_k - \lambda_i^{'}\Delta T \qquad 1$$

where $\sigma_{ij}$ and $\varepsilon_{kl}$ denote the stress and strain tensors, respectively. $E_k$ and $D_m$ are the electric field and electric displacement vectors, respectively. $C_{ijkl}$, $h_{kij}$, $\lambda_{ij}$, $\in_{mk}$, and $\lambda_i^{'}$ state the elastic constants, piezoelectric coefficients, thermal expansion coefficients, dielectric, and pyroelectric, respectively. $e_0a$ is the small scale (nonlocal) parameter based on Eringen's elasticity theory. The constitutive equations for a zigzag structure of the SWBNNT can be written as the following matrix form Eq. (2):

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{21}&C_{22}&C_{23}&0&0&0\\ C_{31}&C_{32}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ 2\varepsilon_{yz}\\ 2\varepsilon_{xz}\\ 2\varepsilon_{xy}\end{bmatrix}-\begin{bmatrix}h_{11}&0&0\\ h_{12}&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}E_x\\ E_y\\ E_z\end{bmatrix}-\begin{bmatrix}\alpha_x\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}\Delta T$$

where $E_x$ is the electric field in the $x$ direction. The electric field versus the electric potential is defined as:

$$E_x = -\frac{\partial\varphi}{\partial x} \qquad 3$$

where $\varphi$ denotes the electric potential function.

Coupled stress theory is first modified by Yang et al. [15]. Therefore, the strain energy based on modified coupled stress theory is given by [1]:

$$U_s = \frac{1}{2}\int_{\Omega} (\sigma_{ij}\varepsilon_{ij} + m_{ij}\hat{\chi}_{ij})dV \qquad i,j = x,y,z \tag{4}$$

where $m_{ij}$ and $\hat{\chi}_{ij}$ denote the deviatoric part of couple stress tensor and rotation gradient symmetric tensor, respectively, that are defined as the following form:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad \hat{\chi}_{ij} = \frac{1}{2}(\beta_{i,j} + \beta_{j,i}) \qquad \beta = \frac{1}{2}e_{ijk}u_{ki} \tag{5}$$

where $u$, $\beta$, and $e_{ijk}$ are the displacement, rotation vectors, and permutation symbol, respectively. The deviatoric part of couple stress tensor is defined as follows:

$$m_{ij} = 2l_0^{\,2}\mu(z)\hat{\chi}_{ij} \tag{6}$$

where $l_0$ and $\mu(z)$ are the material length scale parameter and the shear modulus, respectively.

The displacement fields for Timoshenko beam model can be defined as [1]:

$$U(x,z) = u_o(x) - z\psi(x) \qquad V(x,z) = 0 \qquad W(x,z) = w(x) \tag{7}$$

where $U$, $V$, and $W$ are the components of displacement in $x$, $y$, and $z$, respectively. Also, $\psi$ is the rotation of the cross-section.

Substituting Eq. (7) into Eq. (5) yields:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\left[\frac{\partial \psi}{\partial x}\right] \qquad \gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial w}{\partial x} - \psi \qquad \chi_{xy} = \chi_{yx} = -\frac{1}{4}\left[\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right] \tag{8}$$

Using Eqs. (1), (2), and (6), the components of stress for Timoshenko beam theory can be simplified as the following form:

$$\sigma_x = C_{11}(\varepsilon_x - \alpha_x T) - h_{11}E_x \qquad \sigma_{xz} = C_{55}(2\varepsilon_{xz}) \qquad m_{xy} = C_{55}L_b^2\chi_{xy} + C_{44}L_m^2\chi_{yx} \qquad m_{yx} = C_{55}L_b^2\chi_{xy} + C_{44}L_m^2\chi_{yx} \qquad D_x = h_{11}(\varepsilon_x - \alpha T) + \in_{11} E_x \tag{9}$$

where $L_m$ and $L_b$ are the material constants associated with the field and fibers, respectively [1].

The mixture of rule for FGM can be defined as follows:

$$C = f^rC^r + f^mC^m \qquad f^r + f^m = 1 \qquad f^r = (1-\frac{2z}{h})^n \frac{\rho C^m}{\rho C^m + (1-\rho)C^r} \tag{10}$$

Where $f^m$ and $f^r$ are the volume fraction of the matrix and fibers, respectively.

Using micromechanical model, the FG mechanical, electrical, and thermal properties can be obtained as follows [2]:

$$C_{11}=\frac{(1-\frac{2z}{h})^n\left[\rho C_{11}^r C_{11}^m-(C_{11}^m)^2\rho\right]}{C_{11}^m\rho+C_{11}^r(1-\rho)}+\frac{(C_{11}^m)^2\rho+C_{11}^m C_{11}^r(1-\rho)}{C_{11}^m\rho+C_{11}^r(1-\rho)}$$

$$C_{12}=C_{21}=C_{11}(\frac{\rho C_{12}^r}{C_{11}^r}+(1-\rho)\frac{C_{12}^m}{C_{11}^m})$$

$$C_{22}=\rho C_{22}^r+(1-\rho)C_{22}^m+\frac{C_{12}^2}{C_{11}}+\frac{\rho(C_{12}^r)^2}{C_{11}^r}-\frac{(1-\rho)(C_{12}^r)^2}{C_{11}^m}$$

$$C_{13}=C_{31}=C_{11}(\frac{\rho C_{13}^r}{C_{11}^r}+(1-\rho)\frac{C_{13}^m}{C_{11}^m})$$

$$C_{23}=\rho C_{23}^r+(1-\rho)C_{23}^m+\frac{C_{12}C_{13}}{C_{11}}-\frac{\rho C_{12}^r C_{13}^r}{C_{11}^r}-\frac{(1-\rho)C_{12}^m C_{13}^m}{C_{11}^m}$$

$$C_{33}=\rho C_{33}^r+(1-\rho)C_{33}^m+\frac{C_{13}^2}{C_{11}}-\frac{\rho(C_{13}^r)^2}{C_{11}^r}-\frac{(1-\rho)(C_{13}^m)^2}{C_{11}^m}$$

$$(C_{44}^r-C_{44}^m)=\frac{1}{\rho}(C_{11}-C_{44}^m)$$

$$C_{55}=\frac{(1-\frac{2z}{h})^n\left[\rho C_{55}^r C_{55}^m-(C_{55}^m)^2\rho\right]}{C_{55}^m\rho+C_{55}^r(1-\rho)}+\frac{(C_{55}^m)^2\rho+C_{55}^m C_{55}^r(1-\rho)}{C_{55}^m\rho+C_{55}^r(1-\rho)}$$

$$\frac{1}{C_{66}}=\frac{\rho}{C_{66}^r}+\frac{(1-\rho)}{C_{66}^m}$$

$$\rho(z)=\rho_r(\frac{(1-\frac{2z}{h})^n\rho\rho_m}{\rho\rho_m+(1-\rho)\rho_r})+\rho_m(1-[\frac{(1-\frac{2z}{h})^n\rho\rho_m}{\rho\rho_m+(1-\rho)\rho_r}])$$

$$\in_{11}=\frac{(1-\frac{2z}{h})^n\left[\rho\in_{11}^r\in_{11}^m-(\in_{11}^m)^2\rho\right]}{\in_{11}^m\rho+\in_{11}^r(1-\rho)}+\frac{(\in_{11}^m)^2\rho+\in_{11}^m\in_{11}^r(1-\rho)}{\in_{11}^m\rho+\in_{11}^r(1-\rho)}$$

$$e_{11}=C_{11}(\frac{\rho e_{11}^r E_1^r C_{11}^m+(1-\rho)e_{11}^m E_1^m C_{11}^r}{C_{11}^r C_{11}^m(\rho E_1^r+(1-\rho)E_1^m)})$$

$$\alpha_x=\rho\alpha_x{}^r+(1-\rho)\alpha_x{}^m$$

where

$$\rho=\frac{C_{11}(C_{11}^m-C_{11}^r)}{C_{11}^r(C_{11}^m-C_{11})} \quad 11, 12$$

## 3. The governing equations of motion for FG nanocomposite Timoshenko beam

In the present study, the total potential energy can be defined as follows:

$$\Pi=U-(K+W_{external}) \quad 13$$

where $U$, $K$, and $W_{external}$ denote the strain energy, kinetic energy and the work done due to the external load. The strain energy based on nonlocal FG nanocomposite Timoshenko beam model subjected to electro-thermo-mechanical loadings using MCSTcan be written as:

$$U=\frac{1}{2}\int_0^L\int_A(\sigma_x\varepsilon_x+2\sigma_{xz}\varepsilon_{xz}+2m_{xy}\hat{\chi}_{xy}-D_xE_x)dA\,dx \quad 14$$

The kinetic energy of FG nanocomposite beam is considered as follows:

$$K=\frac{1}{2}\int_0^L\int_A\rho(z)\,[(\frac{\partial u}{\partial t})^2+(\frac{\partial w}{\partial t})^2]dA\,dx \quad 15$$

The work done due to external load is defined as the following form:

$$W_{external}=\int_0^L f\,w\,dA \quad 16a$$

where $f$ including visco-elastic medium and transverse load is written as

$$f=-K_w w+K_p\nabla^2 w-C_d\frac{\partial w}{\partial t}-f_w \quad 16b$$

In which $K_w$, $K_p$, and $C_d$ are the visco-elastic coefficients. Also, $f_w$ denotes the transverse load. Using the minimum principle of total potential energy, we have:

$$\int_{t_0}^{t_1} \delta\Pi\, dt = 0$$
$$\int_{t_0}^{t_1} \delta\left(U - (K + W_{external})\right) dt = 0 \qquad 17$$

Using Eq. (17), substituting Eqs. (3), (8) and (9) into Eq. (14) and employing Eqs. (15) and (16), the governing equations of motion for FG nanocomposite beam are obtained as follows:

$$\delta u_0 : -\frac{\partial N_x}{\partial x} - \frac{1}{2} H_{11}^0 b \frac{\partial^2 \varphi}{\partial x^2} + \ddot{u}\, m_0 = 0 \qquad 18$$

$$\delta\psi : \frac{\partial M_x}{\partial x} - Q + \frac{1}{2}\frac{\partial Y}{\partial x} + m_2 \ddot{\psi} + \frac{b}{2} H_{11}^1 \frac{\partial^2 \varphi}{\partial x^2} = 0$$

$$\delta\varphi : -\frac{1}{2} H_{11}^0 b \frac{\partial^2 u_0}{\partial x^2} + \frac{1}{2}\frac{\partial}{\partial x}\left(H_{11\alpha} bT\right) + E_{11} b \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{2} b H_{11}^1 \frac{\partial^2 \psi}{\partial x^2} = 0$$

$$\delta w : -\frac{\partial Q}{\partial x} - \frac{1}{2}\frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} - f = 0 \qquad (21$$

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where

$$N_X = \int_A \sigma_X\, dA \qquad Q = k_s \int_A \sigma_{XZ}\, dA$$
$$M_X = \int_A z\sigma_x\, dA \qquad Y = \int_A m_{xy}\, dA$$
$$H_{11}^i = \int h_{11} z^i dz \qquad E_{11} = \int \in_{11} dz$$
$$H_{11\alpha} = \int \alpha_x h_{11} dz \qquad 22$$

$$m_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\, dz \qquad m_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \rho(z)\, d. \qquad 23$$

where $k_s$ is the shear correction factor.

To satisfy simply supported boundary conditions, the Navier's type solution is defined as:

$$u_0 = U_{(x)} e^{i\omega t} = A \sin\frac{m\pi x}{L} e^{i\omega t}$$
$$\psi = \psi_{0(x)} e^{i\omega t} = B \sin\frac{m\pi x}{L} e^{i\omega t}$$
$$w = W_{(x)} e^{i\omega t} = C \cos\frac{m\pi x}{L} e^{i\omega t} \qquad 24$$
$$T = T_{(x)} e^{i\omega t} = D \cos\frac{m\pi x}{L} e^{i\omega t}$$
$$f_w = q_0 \sin\frac{m\pi x}{L}$$

where $\omega$ and $m$ are the natural frequency and axial wave number, respectively.

After simplifying the equation and eliminating potential function from Eqs. (18)-(21), we obtained three coupled equations of motion in terms of $u_0$, $\psi$, and $w$. Substituting Eqs. (24) into these equations yields: 18)

$$\left\{[M]\omega^2 + [C]i\omega + [K]\right\}\{U\} = [b] \qquad 25$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices, respectively. $\{U\}$ is the displacement vector which is considered as follows:

$$\{U\} = \begin{Bmatrix} U \\ \psi \\ W \end{Bmatrix} \qquad 26$$

To solve the Eq. (25) and reducing it to the standard form of eigenvalue problem, it is convenient to rewrite Eq. (25) as the following first order variable [14]:

$$\{\dot{U}\} = [A]\{U\} \qquad 27$$

in which the state matrix [A] can be defined as:

$$[A] = \begin{bmatrix} [0] & [I] \\ -[M^{-1}][K] & -[M^{-1}][C] \end{bmatrix} \qquad 28$$

## 4. Results and discussion

In this research, the bending and free vibration analysis of nonlocal FG nanocomposites Timoshenko beam model reinforced by the SWBNNT embedded in an elastic medium based on MCST is investigated. The effects of the slenderness ratio, length of nanocomposite beam, material length scale parameter, nonlocal parameter, power law index, axial wave number, and Winkler and Pasternak coefficients on the natural frequency of nanocomposite beam are taken into account.

The obtained results of this research are compared with the results by Chen et al. [1] for isotropic Timoshenko beam model in Table 1 for the following values:

$$h = b = 25\,\mu m,\; L = 200\,\mu m,\quad E = 174.5\mu m,\; \nu = 0.25,\; q_0 = 1\; N/mm \qquad (29)$$

Table 1 shows that the results of the present work are in the good agreement with those by Chen et al. [1].

**Table 1.** Comparison between the results of present work and the obtained results by Chen et al. [1]

| | $x = 0.25L$ | $x = 0.5L$ | $x = 0.9L$ |
|---|---|---|---|
| $w/h \times 10^9$ (chen et al.[1]) | 1.2735 | 1.8011 | 0.5566 |
| $w/h \times 10^9$ Present work | 1.27354 | 1.80106 | 0.55656 |

The obtained results of the present work are compared with the other results for simply supported isotropic homogeneous Timoshenko micro-beam model based on the modified couple stress theory in Table 2 for the following values:

$$k_s = 5/6,\; L/h = 10,\quad \upsilon = 0.38,\quad \rho = 1220 kg/m^3,\; E = 1.44 GPa,\quad l = 17.6\mu m \qquad (30)$$

A good agreement is found between the present work and the other work results [16,17, 18]. It is seen that the natural frequency increases with decreasing $h/l$.

Fig. 1 shows the influence of material length scale parameter based on MCST on the first natural frequency. It is seen from this figure that the natural frequency diminishes with an increase in the material length scale parameter for low slenderness ratio. In the other hand, with increasing slenderness ratio, the effect of material length scale parameter on the natural frequency is negligible.

**Table 2.** Comparison of first two natural frequencies (MHz) for isotropic homogeneous Timoshenko micro-beam model

| | Present work | Ansari et al. [16] | Ma et al. [17] | Ke and Wang [18] |
|---|---|---|---|---|
| $h/l = 10$, $m = 1$ | 0.037648 | 0.03766 | 0.03780 | 0.03746 |
| $h/l = 10$, $m = 2$ | 0.139659 | 0.13977 | 0.1416 | 0.1390 |
| $h/l = 5$, $m = 1$ | 0.077817 | 0.07788 | 0.07782 | 0.07636 |
| $h/l = 5$, $m = 2$ | 0.288745 | 0.28888 | 0.2887 | 0.2837 |
| $h/l = 3.33$, $m = 1$ | 0.122899 | 0.1229 | 0.1227 | 0.1180 |
| $h/l = 3.33$, $m = 2$ | 0.456088 | 0.4561 | 0.4555 | 0.4393 |

Fig. 2. illustrates the influence of nanocomposite beam length on the first natural frequency. It is concluded that with increasing of the length of nanocomposite beam, the natural frequency decreases.

Fig. 3. shows the influence of axial wave number on the natural frequency. It is seen that the first natural frequency increases with an increase in the values of axial wave number.

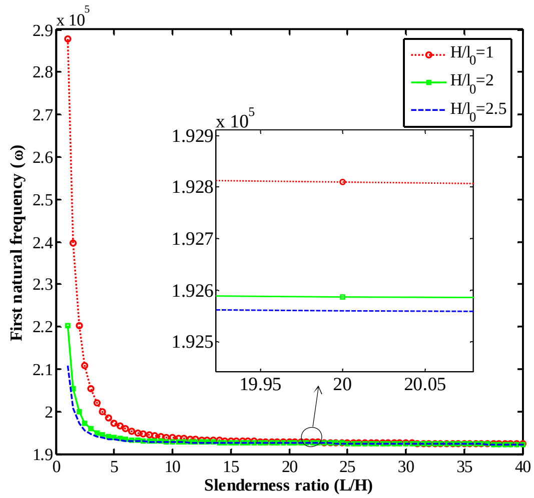


**Fig. 1.** The influence of material length scale parameter on the first natural frequency.

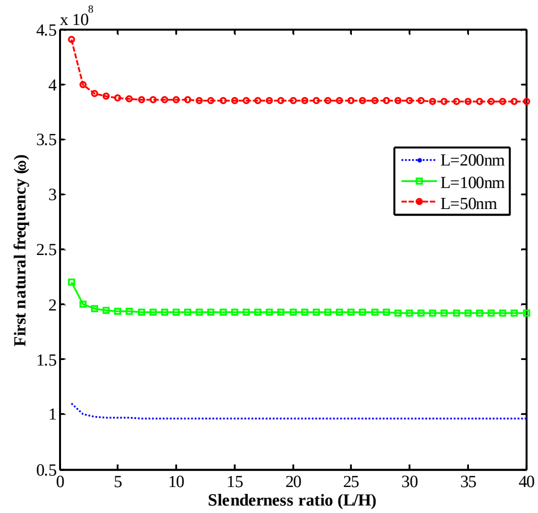


**Fig. 2.** The influence of nanocomposite beam length on the first natural frequency.

Fig. 4 shows the influence of nonlocal parameter on the natural frequency. It can be seen that with increasing of the nonlocal parameter, the natural frequency of FG nanocomposite beam model reduces.

The effect of the elastic medium on the natural frequency is shown in Fig. 5. It is seen that for low slenderness ratio, a change of theelastic medium leads to increase the natural frequency of FG nanocomposite beam. It is important to note that the elastic medium makes the FG nanocomposite beam model to be stiffer than without considering the elastic medium. In addition, the natural frequency of the visco-Winkler-Pasternak model is higher than that of the other models.

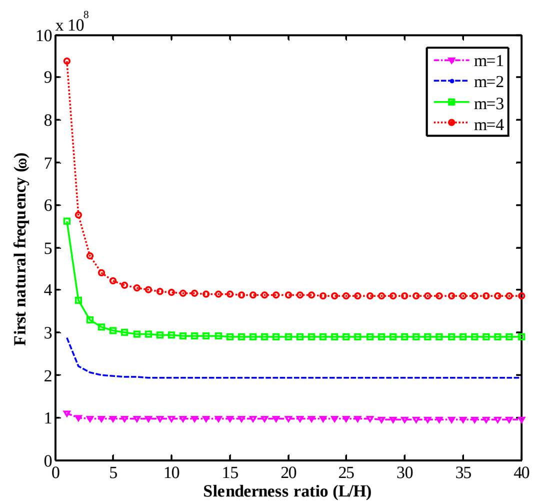


**Fig. 3.** The influence of axial wave number on the first natural frequency.

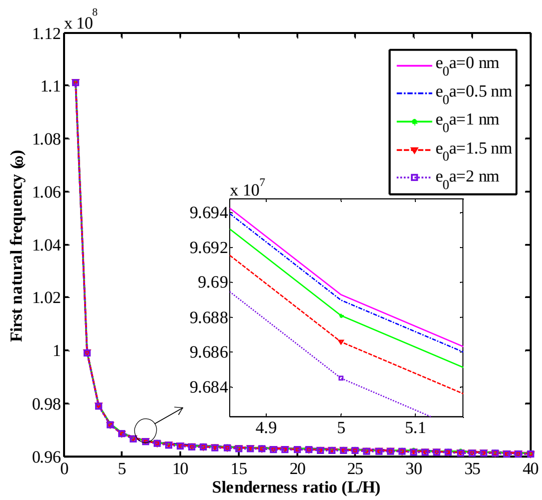


**Fig. 4.** The influence of nonlocal parameter on the natural frequency.

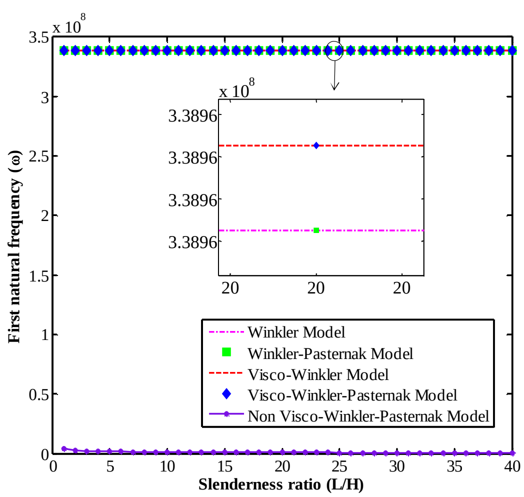


**Fig.5.** The influence of elastic medium coefficient on the natural frequency.

Fig. 6 illustrates the influence of power law index on the natural frequency. It is seen that for low slenderness ratio, the natural frequency decreases with increasing of the power law index.

Fig. 7 shows the influence of material length scale parameter on the dimensionless deflection of FG nanocomposite Timoshenko beam model for simply supported boundary conditions. It can be seen that the dimensionless deflection of FG nano beam increases with an increase in thematerial length scale parameter.

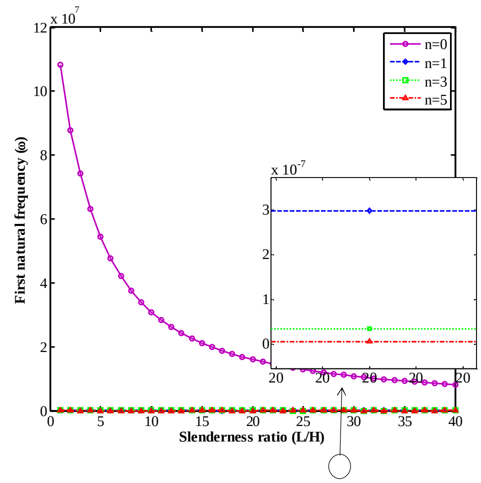


**Fig. 6.** The influence of power law index on the natural frequency.

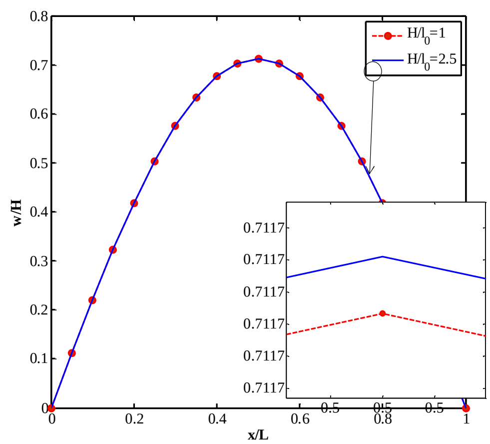


**Fig. 7.** The influence of material length scale parameter on the dimensionless deflection of FG nanocomposite Timoshenko beam model

## 5. Conclusion

In this work, the bending and free vibration analysis of FG nanocomposites Timoshenko beam model using micro-mechanical approach reinforced by SWBNNTs embedded in an elastic medium was studied. To consider the size-dependent effect, MCST and nonlocal elasticity theory were used in this article. The influences of the slenderness ratio, length of nanocomposite beam, material length scale parameter, nonlocal parameter, power law index, axial wave number, and Winkler and Pasternak coefficients on the natural frequency of nanocomposite beam were investigated. Moreover, the effect of material length scale parameter on the dimensionless deflection of FG nanocomposite beam was presented. The obtained results of the present work are compared with the other results [16, 17, 18] for simply supported isotropic homogeneous Timoshenko micro-beam model based on the modified couple stress theorythat there are a good agreement between them. The following results were obtained:

1-The natural frequency diminishes with an increase in the material length scale parameter for low slendernessratio. 2- With increasing the length of nanocomposite beam, the natural frequency decreases. 3- The first natural frequency increases with an increase in the values of axial wave number. 4- With increasing the nonlocal parameter, the natural frequency of FG nanocomposite reduces. 5-For low slenderness ratio, a change of the Winkler - Pasternak coefficients with respect to Winkler coefficient leads to increase in the natural frequency. Also, the natural frequency for the visco-Winkler-Pasternak model is higher than that of the other models. 6- The elastic medium causes that the FG nanocomposite beam model becomes stiffer. 7- For low slenderness ratio, the natural frequency decreases with increasing of the power law index.

8- The dimensionless deflection of FG nano beam increases with increasing of the material length scale parameter.

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