

## Vibration of Piezoelectric Nanowires Including Surface Effects

R. Ansari\*, M. A. Ashrafi, S. Hosseinzadeh, Z. Firouznia

Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran

### Article history:

Received 2/3/2014

Accepted 19/5/2014

Published online 1/6/2014

### Keywords:

Piezoelectric nanowires

Vibration behavior

Timoshenko beam model

Surface effects

### \*Corresponding author:

E-mail address:

r\_ansari@guilan.ac.ir

Phone: 98 1316690276

### Abstract

In this paper, surface and piezoelectric effects on the vibration behavior of nanowires (NWs) are investigated by using a Timoshenko beam model. The electric field equations and the governing equations of motion for the piezoelectric NWs are derived with the consideration of surface effects. By the exact solution of the governing equations, an expression for the natural frequencies of NWs with simply-supported boundary conditions is obtained. The effects of piezoelectricity and surface effects on the vibrational behavior of Timoshenko NWs are graphically illustrated. A comparison is also made between the predictions of Timoshenko beam model and those of its Euler-Bernoulli counterpart. Additionally, the present results are validated through comparison with the available data in the literature.

2014 JNS All rights reserved

## 1. Introduction

In recent years, NWs have attracted a lot of attention for a diversity of technological applications in biotechnology, electronics and photonics as sensors, actuators, transistors, probes and resonators [1-3]. Existing results reveal that the mechanical behavior of nanostructures is size-dependent. Because of high surface-to-bulk ratio, the physical and chemical properties of nanomaterials are affected by surface effects. In this regard, so many endeavors have been made to understand the influence of surface effects on nanostructures. For example, the effects of surface elasticity on the resonance frequency of

nanobeams were studied by Gurtin et al. [4] and Lu et al. [5]. Miller and Shenoy [6] investigated the stretching and bending problems of nanosized structural elements. The effects of both surface stress and surface elasticity on the vibration of nanobeams were studied by Wang and Feng [7]. Ansari and Sahmani [8] studied bending behavior and buckling of nanobeams with the consideration of surface effects for different types of beam theories.

In addition, much effort has been performed for studying the surface effects on mechanical properties of NWs. Cuenot et al. [9] have displayed that the stiffness of NWs is size-dependent. By

using Euler–Bernoulli beam theory, He and Lilley [10] demonstrated the surface effects on the elastic behavior of NWs. Park [11] studied the surface stress on the resonant frequencies of silicon NWs with finite deformation using the surface Cauchy–Born model.

On most existing studies, NWs with a simple Euler–Bernoulli beam model were treated and the shear deformation and rotary inertia effects were neglected. The Euler–Bernoulli beam model can be utilized only for beam length-to-thickness ratios of the order 20 or more. In many applications, the length of NWs is not sufficiently large and utilizing the Euler–Bernoulli beam model is not accurate. Hence, for the NWs with small length-to-thickness ratios, employing the Timoshenko beam model is essential to catch the effects of rotary inertia and shear deformation. Recently, the surface effects on the buckling and vibration behavior of NWs were studied using the Timoshenko beam model. Using a comprehensive Timoshenko beam model, Jiang and Yan [12] investigated surface effects on the static bending of NWs. By using the Timoshenko beam theory, Wang and Feng [13] studied surface effects on the axial buckling and the transverse vibration of NWs. Hasheminejad and Gheshlaghi [14] investigated dissipative surface stress effects on free vibrations of NWs. In addition, this type of beam theories was applied in vibration and buckling behavior of other nanostructures [15-17].

Piezoelectric nanostructures are being developed to convert nanoscale mechanical energy into electric energy [18,19]. Recently, piezoelectric NWs have attracted much attention due to their applications as diodes, nanoresonators and nanogenerators [20,21]. Direct electricity was generated from piezoelectric NWs [22-24]. The electrostatic potential in a bending piezoelectric

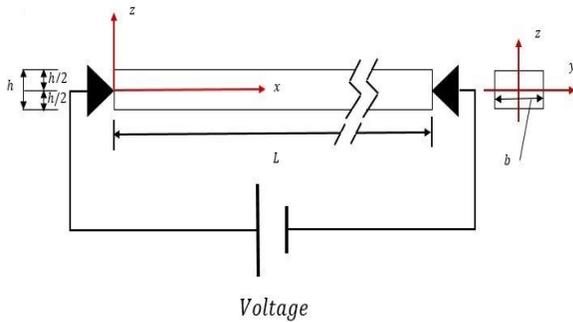
NW was calculated using the perturbation theory [25] and finite element method [26]. Studying the size-dependent behavior of piezoelectric NWs with surface effects has been reported in the literature. Gheshlaghi and Hasheminejad [27] presented an analytical model for predicting surface effects on the free vibrations of piezoelectric NWs based on the Euler-Bernoulli beam theory. Yan and Jiang [28] investigated the electromechanical coupling behavior of piezoelectric NWs with consideration of surface effects and surface piezoelectricity using Euler beam theory. Using Timoshenko beam theory, Samaei et al. [29] studied the buckling behavior of piezoelectric NWs under distributed transverse loading. By using the Euler–Bernoulli beam model, Wang and Feng [30] studied the effects of surface stresses on the vibration and buckling of piezoelectric NWs.

Hosseini-Hashemi et al. [34] investigated surface effects on the free vibrations of piezoelectric nanobeam based on the nonlocal Euler-Bernoulli beam model.

In this paper, an analytical method is presented to study the combined piezoelectric and surface effects on the vibrational behavior of piezoelectric NWs using the Timoshenko beam model. The electric field equations and the governing equations of motion are obtained and then, for NWs with simply-supported end condition, an exact expression of natural frequency is derived. The influence of piezoelectricity and surface effects on the vibrational behavior of piezoelectric NWs have been demonstrated. For a specific size of NW, the numerical results of the Timoshenko beam model are compared with those of the Euler-Bernoulli beam model.

## 2. Overview of Timoshenko beam theory

Consider a simply-supported piezoelectric NW with length  $L$ , width  $b$  and thickness  $h$  as shown in Fig. 1 and assume the deformation of this NW takes place in the  $x - z$  plane. So,  $u_x$ ,  $u_y$  and  $u_z$  are the components along the axis  $x$ ,  $y$  and  $z$  of NW displacement vector, respectively.



**Fig. 1.** Piezoelectric NW with rectangular cross section and its coordinate system

The displacement field for a Timoshenko beam can be described by

$$\begin{aligned} u_x &= z\psi(x,z) \\ u_y &= 0 \\ u_z &= w(x,z) \end{aligned} \tag{1}$$

where  $w(x,z)$  and  $\psi$  denote the lateral deflection of the beam and the rotation of beam cross section, respectively.

Also, the nonzero components of the strain tensor for this beam are given by

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} = z \frac{\partial \psi}{\partial x}$$

$$\gamma_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = \frac{1}{2} \left( \psi + \frac{\partial w}{\partial x} \right) \tag{2}$$

## 3. Surface and piezoelectric effects

### 3.1. Surface effects

The influence of surface energy or surface stresses on the mechanical behavior of nanomaterials is usually mentioned as surface effects.  $\sigma_{\alpha\beta}^s$  denotes the surface stress tensor which

is related to the surface energy density  $\gamma$  and is given by [31]

$$\sigma_{\alpha\beta}^s = \gamma \delta_{\alpha\beta} + \frac{\partial \gamma}{\partial \epsilon_{\alpha\beta}^s} \tag{3}$$

where  $\epsilon_{\alpha\beta}^s$  and  $\delta_{\alpha\beta}$  indicate the surface strain tensor and the Kronecker delta, respectively.

The linear form of Eq. (3) is expressed as

$$\sigma^s = \tau^0 + E^s \epsilon \tag{4}$$

in which  $\tau^0$  and  $E^s$  are the residual surface tension under unstrained condition and the surface elastic modulus, respectively. These constants are specified by atomistic simulations or experimental measurements.

### 3.2. Piezoelectric effect

The linear constitutive equations for a homogeneous orthotropic piezoelectric material are given by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{12} & E_{11} & E_{13} & 0 & 0 & 0 \\ E_{13} & E_{12} & E_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_1 & 0 \\ 0 & 0 & 0 & e_4 & 0 & 0 \\ e_2 & e_3 & e_3 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \\ 0 & 0 \end{bmatrix} \quad (6)$$

where  $E_{ij}$ ,  $e_i, \lambda_i$  indicate the matrices of elastic constants, piezoelectric constants and dielectric constants, respectively.

In the piezoelectric materials, the electric-field components are specified by the electric potential  $\Phi$  as follows

$$\varphi_x = -\frac{\partial \Phi}{\partial x}, \quad \varphi_z = -\frac{\partial \Phi}{\partial z}, \quad \varphi_y = 0 \quad (7)$$

The poling direction of the piezoelectric medium is along the positive x-axis, where (x, z) is a rectangular Cartesian coordinate system as shown in Fig 1.

From numerical simulations which were reported in the literature, it can be concluded that for a piezoelectric NW subjected to bending, the electric potential along the NW (x-axis) except in the vicinity of two ends, is almost constant. So, the electric-field components satisfy  $\varphi_x \ll \varphi_z$  [18].

For a piezoelectric NW, the nonzero electric displacements components and the nonzero stress components are obtained From Eqs. (5) and (6) and are given by

$$\begin{aligned} D_x &= e_1 \gamma_{yz} + \lambda_{11} \varphi_x \\ D_z &= e_2 e_{zz} + \lambda_{22} \varphi_z \end{aligned} \quad (8)$$

$$\begin{aligned} \sigma_{zz} &= E_{11} e_{zz} - e_2 \varphi_z \\ \sigma_{xz} &= E_{12} \gamma_{yz} - e_3 \varphi_x \end{aligned} \quad (9)$$

in which, dielectric constants  $\lambda_{11}$  and  $\lambda_{22}$  are on the same order and  $\varphi_x \ll \varphi_z$ , so the electric displacement  $D_x$  is insignificant in comparison with  $D_z$ .

Electrostatic equilibrium condition in the absence of electric charges is given by [30]

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0 \quad (10)$$

### 4. Derivation of the electric field equations and the governing equations of motion

#### 4.1. Electric field equations

Using the Timoshenko beam model, substitution of Eqs. (1), (2), (7) and (8) into (10) leads to electric field equation as

$$\Phi = \left[ \left( \frac{e_2 + 2e_3}{4\lambda_{22}} \right) \psi' + \left( \frac{e_2}{4\lambda_{22}} \right) w'' \right] z^2 + c_1 z + c_2 \quad (11)$$

in which a prime represents the derivative with respect to  $x$ .

Assuming  $\Phi \left( -\frac{h}{2} \right) = 0$  and  $\Phi \left( \frac{h}{2} \right) = 2V$  leads

to the following equation for electric field

$$\Phi = \left( \frac{e_2}{4\lambda_{22}} + \frac{e_3}{2\lambda_{22}} \right) \left( z^2 - \frac{h^2}{4} \right) \psi' + \left( \frac{e_2}{4\lambda_{22}} \right) \left( z^2 - \frac{h^2}{4} \right) w'' \quad (12)$$

In addition, the nonzero components of stress tensor for this beam model can be obtained from Eqs. (7), (9) and (12), as

$$\begin{aligned} \sigma_{zz} &= E z (\psi') + z \left( \frac{e_2 e_{zz}}{2\lambda_{22}} + \frac{e_3^2}{\lambda_{22}} \right) (\psi') + z \left( \frac{e_2 e_{zz}}{2\lambda_{22}} \right) (w'') + 2e_3 \frac{V}{h} \\ \sigma_{xz} &= \sigma_{zx} = k_s \mu (\psi + w) \end{aligned} \quad (13)$$

where  $k_s$  is the shear coefficient which for rectangular cross-section is given by

$$k_s = \frac{E(1+\nu)}{(6+5\nu)} \tag{14}$$

### 4.2. Governing equations of motion

For a deformed beam, the presence of residual surface stress leads to generation of a distributed normal pressure  $q$ . This pressure is along the longitudinal direction and depends on the curvature of the bending beam  $w''$  [12,13]

$$q = Hw'' \tag{15}$$

where  $w''$  indicates the curvature of the beam and  $H$  is a constant relevant to the residual surface stress and the cross-sectional shape which for a rectangular cross-section with width  $b$  is given by [12,13]

$$H = 2b\tau^2 \tag{16}$$

The influence of the second term in Eqs. (4) and (9), for a rectangular cross-section NW can be represented by the effective flexural rigidity  $(EI)_{eff}$  as

$$EI_{eff} = EI + E^* \left( \frac{bh^3}{2} \right) + E^* \left( \frac{h^3}{6} \right) + I \left( \frac{2\varepsilon_1^2 + \varepsilon_2\varepsilon_3}{2\lambda_{33}} \right) \tag{17}$$

where  $E$  is Young's modulus and  $I$  is inertia moment which for a rectangular cross-section is given by  $\frac{1}{12}bh^3$ .

On a cross-section of the NW, the bending moment  $M$  and shear force  $Q$  are given by

$$M = \int \sigma_{xx} y dA$$

$$Q = \int \sigma_{xz} dA \tag{18}$$

The Euler-Lagrange equations for Timoshenko beam model are obtained in the form

$$Q' + q - Pw' = 0$$

$$M - Q = 0 \tag{19}$$

where  $P$  is the resultant axial force which is generated by the applied electric potential and is defined in the form

$$P = b \int_{-h/2}^{h/2} \sigma_{xx} ds = 2Vb\varepsilon_1 \tag{20}$$

Substituting Eqs. (15) and (18) into (19) gives the following equilibrium equations

$$k_s b A (w'' + \psi) + 2b(\varepsilon_2 - \nu\varepsilon_3)(w') - \frac{1}{6} \left( \frac{\varepsilon_2 \varepsilon_3 I}{\lambda_{33}} \right) (\psi') = \rho$$

$$(EI)_{eff} (\psi') + I \left( \frac{\varepsilon_2 \varepsilon_3}{2\lambda_{33}} \right) (w') - k_s b A (w' + \psi) = \rho \dot{w} \tag{21}$$

where a dot represents the derivative with respect to time.

### 5. Free vibrations of the simply-supported NWs

In this section, Navier solution of the governing equations is presented for the piezoelectric NW with simply-supported end conditions. This method is utilized for the Timoshenko beam model and the exact expression of natural frequencies is obtained.

For this beam model, the periodic solutions are assumed in the following form

$$w(x,t) = c_1 \sin \left( \frac{n\pi x}{L} \right) e^{i\omega t}$$

$$\psi(x,t) = c_2 \cos \left( \frac{n\pi x}{L} \right) e^{i\omega t} \tag{22}$$

where  $\omega$  is the frequency of free vibration.

Substitution of Eq. (21) into (22) and rearranging them in a matrix form gives

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
 a_{11} &= \frac{1}{4} \frac{\epsilon_1 \epsilon_2 I n^2 \pi^2}{L^2 \lambda_{33}} - \frac{k_3 u A n \pi}{L} \\
 a_{12} &= -\frac{(EI)_{eff} n^2}{L^3} - k_3 u A + \rho I \omega^2 \\
 a_{21} &= -\frac{k_3 u A n^2 \pi^2}{L^3} - \frac{2 b n^2 \pi^2 \tau_0}{L^3} + \frac{2 b n^2 \pi^2 V \epsilon_1}{L^3} + \rho A \omega^2 \\
 a_{22} &= -\frac{1}{4} \frac{\epsilon_1 \epsilon_2 I n^2 \pi^2}{L^2 \lambda_{33}} - \frac{k_3 u A n \pi}{L} \quad (23)
 \end{aligned}$$

For obtaining non-trivial solution of Eq. (23), the determinant of the coefficient matrix must be equal to zero which results a characteristic equation. The eigenvalues of the obtained characteristic equation are the n-th natural frequencies of vibration for a piezoelectric NW with simply-supported end conditions. The expression of fundamental natural frequency for Timoshenko NW is obtained in the form of

$$\begin{aligned}
 \omega = & \\
 & -\frac{1}{2} \frac{1}{\lambda_{33} I A L} \left( \sqrt{2} \left( \lambda_{33} \rho I A \left( \lambda_{33} k_3 u A^2 L^2 + \right. \right. \right. \\
 & \lambda_{33} (EI)_{eff} n^2 A + \lambda_{33} I k_3 u A n \pi^2 + 2 \lambda_{33} I b n^2 \tau_0 - \\
 & 2 \lambda_{33} I b n^2 V \epsilon_1 + \left. \left. \left( \lambda_{33}^2 k_3^2 u^2 A^4 L^4 + 4 \lambda_{33}^2 I^2 b^2 n^4 \tau_0^2 + \right. \right. \right. \\
 & \left. \left. \left. \lambda_{33}^2 (EI)_{eff}^2 n^4 A^2 + 2 \lambda_{33}^2 k_3 u A^2 L^2 (EI)_{eff} n^2 + \right. \right. \right. \\
 & \left. \left. \left. 2 \lambda_{33}^2 k_3^2 u^2 A^3 L^2 I \pi^2 - 2 \lambda_{33}^2 (EI)_{eff} n^4 A^2 I k_3 u - \right. \right. \right. \\
 & \left. \left. \left. 4 \lambda_{33}^2 (EI)_{eff} n^4 I b \tau_0 - 8 \lambda_{33}^2 I^2 b^2 n^4 \tau_0 V \epsilon_1 + \right. \right. \right. \\
 & \left. \left. \left. \lambda_{33}^2 I^2 k_3^2 u^2 n^2 \pi^4 A^2 + 4 \lambda_{33}^2 I^2 b^2 n^4 V^2 \epsilon_1^2 - \right. \right. \right. \\
 & \left. \left. \left. 4 \lambda_{33}^2 k_3 u A^2 L^2 I b n^2 \tau_0 + 4 \lambda_{33}^2 k_3 u A^2 L^2 I b n^2 V \epsilon_1 + \right. \right. \right. \\
 & \left. \left. \left. 4 \lambda_{33}^2 (EI)_{eff} n^4 I A b V \epsilon_1 + 4 \lambda_{33}^2 I^2 k_3 u A n^4 b \tau_0 - \right. \right. \right. \\
 & \left. \left. \left. 4 \lambda_{33}^2 I^2 k_3 u A n^4 b V \epsilon_1 + 4 \lambda_{33}^2 I^2 A^2 k_3 u n^4 \epsilon_1 \epsilon_2 \right) \right)^{\frac{1}{2}} \right) \quad (24)
 \end{aligned}$$

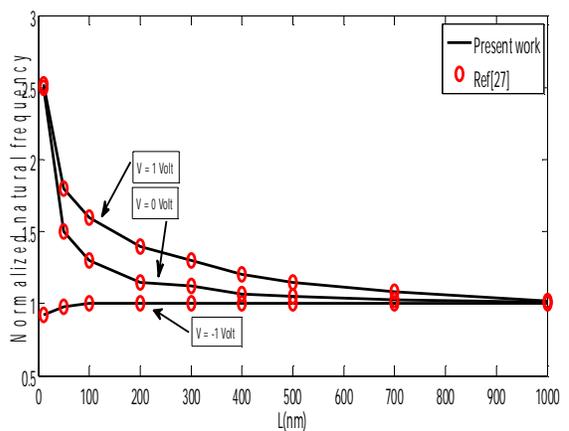
**6. Results and discussion**

Here, the numerical results are given to illustrate the influence of piezoelectricity on the vibration

behavior of the piezoelectric NW. consider a simply-supported piezoelectric NW which its material and physical properties are as:  $b = h$ ,  $\nu = 0.3$ ,  $\tau_0 = 0.5$ ,  $\lambda_{33} = -7.88 \times 10^{-11}$ ,  $E = 207 \times 10^9$ ,  $E^s = 0$ ,  $\mu = E/2(1 + \nu)$ ,  $\epsilon_1 = -0.51$ ,  $\epsilon_2 = -0.45$ .

The natural frequency is normalized with respect to the fundamental frequency of classical Euler-Bernoulli beam model as  $\omega_0 = \left(\frac{h}{L}\right)^2 \sqrt{\frac{EI}{\rho h^3}}$  and is obtained for the first mode ( $n = 1$ ).

The results of present study can be verified through comparison with the ones reported in Ref. [27] on the basis of the Euler-Bernoulli beam theory as shown in Fig. 2, at  $L/h = 20$  for  $(\epsilon_0 a)^2 = 0$   $L/h = 20$ , which  $\epsilon_0$  and  $a$  are material constant and internal characteristic length, respectively.



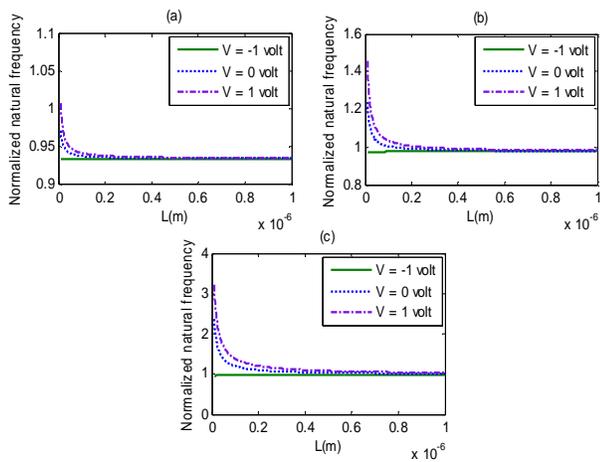
**Fig. 2.** Variation of the normalized fundamental natural frequency of Euler-Bernoulli simply-supported NW.

with length  $L$  for different values of voltage ( $L/h = 20$ ).

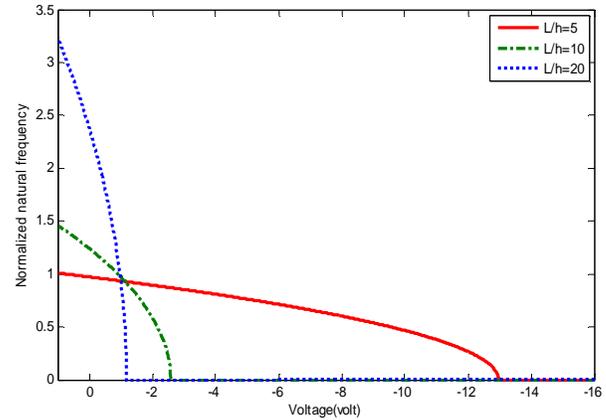
Also, to show the accuracy of the present analysis, normalized natural frequencies obtained from the present analysis are compared with those given in Refs. [33, 34]. Table 1 shows this comparison for different values of voltage.

**Table 1.** Comparison between the results of present work and the obtained results from Refs. [33, 34] ( $L/h = 10$ )

Voltage	Euler-Bernoulli			Timoshenko	
	Ref. [33]	Ref. [34]	Present	Ref. [33]	Present
$V = 1$	1.02	-	1.028	0.98	0.977
$V = 0.5$	-	0.71	0.74	-	-
$V = 0$	1.03	0.99	0.98	0.985	0.981
$V = -0.5$	-	1.19	1.187	-	-
$V = -1$	1.05	-	1.2	0.996	0.99



**Fig. 3.** Variation of the normalized fundamental natural frequency of Timoshenko simply-supported NW with length  $L$  for different values of voltage (a)  $L/h = 5$ , (b)  $L/h = 10$ , (c)  $L/h = 20$



**Fig. 4.** Variation of the normalized fundamental natural frequency of Timoshenko simply-supported NW with voltage  $V$  for different values of  $L/h$ .

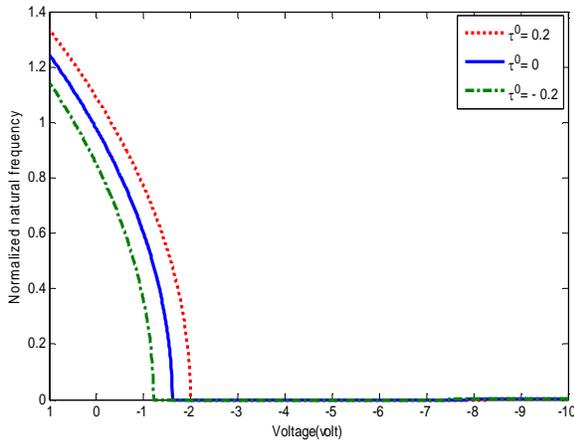
Fig. 3 gives the variation of the normalized fundamental natural frequency of the Timoshenko piezoelectric simply-supported NW with its length ( $L$ ) for selected input voltages ( $V = 0, \pm 1$  volts) and for different values of the

length-to-thickness ratio. It can be found from these figures that by increasing the NW length, piezoelectricity effects gradually vanish. Also, the fundamental natural frequency increases by the increment in the length-to-thickness ratio  $\frac{L}{h}$ . It is

clear that decreasing the input voltage leads to decrease in the NW fundamental natural frequencies. For the positive values of voltage, increasing the length leads to decrease in the fundamental natural frequencies to a constant value and for the negative values of voltage, the fundamental natural frequencies are almost constant. When there is no applied voltage, as the NW length increases, the fundamental natural frequency decreases. Fig. 4 demonstrates the variation of the normalized fundamental natural frequency of the Timoshenko piezoelectric

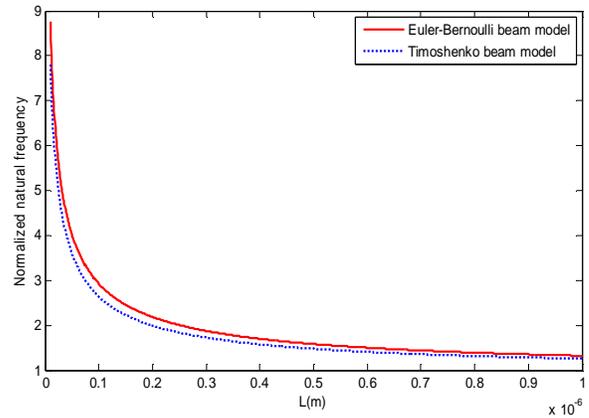
simply-supported NW with voltage for different values of  $\frac{L}{h}$ . The figure displays that for various

values of length-to-thickness ratio, under a specific voltage, the fundamental natural frequency becomes zero and the buckling occurs. The value of this voltage increases as the value of length-to-thickness ratio increases.

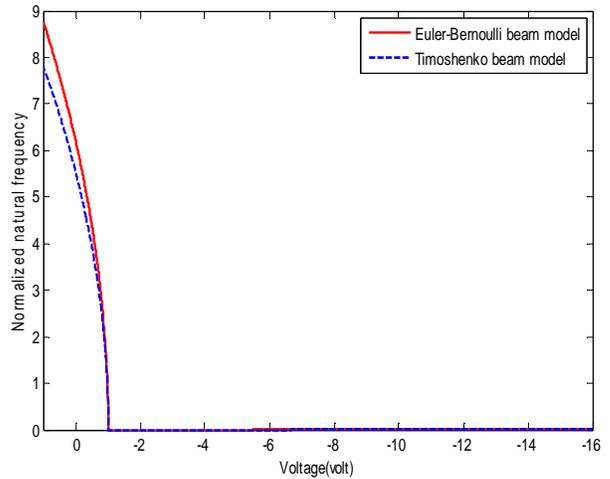


**Fig. 5.** Variation of the normalized fundamental natural frequency of Timoshenko simply-supported NW with voltage  $V$  for different values of  $\tau^0$  ( $L/h = 10$ ).

Fig. 5 displays the variation of the normalized fundamental natural frequency of the Timoshenko piezoelectric simply-supported NW with voltage for different values of the residual surface tension  $\tau^0$ . It can be observed that by increasing the value of  $\tau^0$ , the stiffness of NW will be increased.



**Fig. 6.** Variation of the normalized fundamental natural frequency of Timoshenko and Euler-Bernoulli simply-supported NWs with length  $L$  ( $L/h = 40$ ).



**Fig. 7.** Variation of the normalized fundamental natural frequency of Timoshenko and Euler-Bernoulli simply-supported NWs with voltage  $V$  ( $L/h = 40$ ).

Figs. 6 and 7 demonstrate the variation of the normalized fundamental natural frequency of the Timoshenko and Euler-Bernoulli piezoelectric simply-supported NWs with length and voltage, respectively. These figures show the comparison between the two beam models and reveal the excellent agreement between them. The difference between the frequencies of the Timoshenko and

Euler-Bernoulli beam models is negligible and for both beam models, in the same voltage, the normalized fundamental natural frequency becomes zero and the buckling happen.

## 7. Conclusion

In this paper, the Timoshenko beam model was applied for vibration analysis of piezoelectric NWs with surface effects. First, the electric-field equations were obtained and used to derive the nonzero stress tensor components. The governing equations of motion and the exact expression of the natural frequencies were derived with consideration of piezoelectricity and surface effects. The normalized natural frequency for the first mode of vibration was obtained and utilized to illustrate the influence of piezoelectricity for Timoshenko NWs. Besides, through the exact solution for NWs with simply-supported boundary condition, the values of buckling voltages were illustrated. Based on the results it can be concluded that by increasing the value of the voltage or decreasing the length of NW, the fundamental natural frequency increases. For a specified value of length-to-thickness ratio, the obtained results for this beam model were compared with those of the Euler–Bernoulli beam. The result showed that the fundamental natural frequencies of the Euler–Bernoulli NWs are close to the Timoshenko NWs and the difference between them is insignificant. In addition, it can be found that surface effects and piezoelectricity can significantly affect the frequency of piezoelectric NW. Additionally, the obtained results were compared with the existing results in the literature and the accuracy of the present method was shown.

## References

- [1] G. Wu, H. Ji, K. Hansen, T. Thundat, R. Datar, R. Cote, M.F. Hagan, A.K. Chakraborty, A. Majumdar, A Proc. Natl. Acad. Sci. USA 98 (2001) 1560-1564.
- [2] Y. Cui, Z.H. Zhong, D.L. Wang, W.U. Wang, C.M. Lieber, Nano Lett. 3 (2003) 149-152.
- [3] Z.H. Zhong, D.L. Wang, Y. Cui, M.W. Bockrath, C.M. Lieber, Sci. 302 (2003) 1377-1379.
- [4] M.E. Gurtin, X. Markenscoff, R.N. Thurston, Appl. Phys. Lett. 29 (1976) 529-530.
- [5] P. Lu, H. P. Lee, C. Lu, S. O’Shea, J Phys. Rev. B 72 (2005) 085405.
- [6] R.E. Miller, V. B Shenoy, Nanotechnology 11 (2000) 139-147.
- [7] G.F. Wang, X.Q. Feng, Appl. Phys. Lett. 90 (2007) 231904.
- [8] R. Ansari, S. Sahmani, Int. J. Eng. Sci. 49 (2011) 1244–1255.
- [9] S. Cuenot, C. Frétygny, S. Demoustier-Champagne, B. Nysten, Phys. Rev. B 69 (2004) 165410.
- [10] J. He, C.M. Lilley, Nano Lett. 8 (2008) 1798-1802.
- [11] H.S. Park, Nanotechnology 20 (2009) 115701.
- [12] L.Y. Jiang, Z. Yan, Physica E 42 (2010) 2274–2279.
- [13] G.F. Wang, X.Q. Feng, J. Phys. D: Appl. Phys. 42 (2009) 155411-155415.
- [14] S.M. Hasheminejad, B. Gheshlaghi, Appl. Phys. Lett. 97 (2010) 2531031-2531033.
- [15] C.M. Wang, Y.Y. Zhang, S.S. Ramesh, S. Kitipornchai, J. Phys. D: Appl. Phys. 39 (2006) 3904-3909.
- [16] Y.Y. Zhang, C.M. Wang, V.B.C. Tan, J. Eng. Mech. 132 (2006) 952-958.

- [17] J.C. Hsu, H.L. Lee, W.J. Chang, *Nanotechnology* 18 (2007) 285503.
- [18] Z.L. Wang, J.H. Song, *Sci.* 312 (2006) 242-246.
- [19] P.X. Gao, J.H. Song, J. Liu, Z.L. Wang, *Adv. Mater.* 19 (2007) 67-72.
- [20] X.D. Bai, P.X. Gao, Z.L. Wang, E.G. Wang, *Appl. Phys. Lett.* 82 (2003) 4806-4808.
- [21] J.H. He, C.L. Hsin, J. Liu, L.J. Chen, Z.L. Wang, *Adv. Mater.* 19 (2007) 781-784.
- [22] J.H. Song, J. Zhou, Z.L. Wang, *Nano. Lett.* 6 (2006) 1656-1662.
- [23] X.D. Wang, J.H. Song, J. Liu, Z.L. Wang, *Sci.* 316 (2007) 102-105.
- [24] W.S. Su, Y.F. Chen, C.L. Hsiao, L.W. Tu, *Appl. Phys. Lett.* 90 (2007) 0631101.
- [25] H.J. Xiang, J.L. Yang, J.G. Hou, Q.S. Zhu, *Appl. Phys. Lett.* 89 (2006) 223111.
- [26] C. Li, W.L. Guo, Y. Kong, H.J. Gao, *Appl. Phys. Lett.* 90 (2007) 033108.
- [27] B. Gheshlaghi, S. M. Hasheminejad, *Current. Appl. Phys.* 12 (2012) 1096-1099.
- [28] Z. Yan, L. Jiang, *J Phys D: Appl. Phys.* 44 (2011) 075404.
- [29] A.T. Samaei, M. Bakhtiari, G.F. Wang, *Nanoscale Research Lett.* 7 (2012) 201.
- [30] G. F. Wang, X. Q. Feng, *EPL* 91 (2010) 56007.
- [31] R.C. Cammarata, *Prog.Surf.Sci.* 46 (1994) 1-38.
- [32] K.M. Liew, J.Z. Zhang, C. Li, S.A. Meguid, *Int. J. Sol. Struct.* 42 (2005) 4239-4257.
- [33] A. Tourki Samaei, B. Gheshlaghi, G.F. Wang, *Curr. Appl. Phys.* 13 (2013) 2098-2102.
- [34] S. Hosseini-Hashemi, I. Nahas, M. Fakher, R. Nazemnezhad, *Acta Mech.* 225 (2014) 1555-1564.