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Embedding of a 2D Graphene System in Non-Commutative Space

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Abstract

The BFT approach is used to formulate the electronic states in graphene through a non-commutative space in the presence of a constant magnetic field B for the first time. In this regard, we introduce a second class of constrained system, which is not gauge symmetric but by applying BFT method and extending phase space, the second class constraints converts to the first class constraints so the system becomes a gauge symmetric.

1. Introduction

Graphene is magical material of the 21th century and basically it is a flat monolayer of carbon atoms tightly packed into a two-dimensional (2D) honeycomb lattice [1].

From the electronic point of view, it is either a zero-overlap semimetal or a zero-gap semiconductor, where the conduction and the valence bands are no longer separated by an energy gap [2]. On the other hand, electrons in graphene behave like photons or other ultra-relativistic particles (such as neutrinos), with an energy-independent velocity v_F that is approximately 300 times smaller than the speed of light [3].

Formally, their quantum-mechanical behavior is no longer described in terms of a (non-relativistic) Schrödinger equation and the Dirac equation must be used for the massless neutrinos instead [1, 3, 4]. However, the connection to relativistic physics is even deeper than this because the Hamiltonian for the particles near the K (or Dirac points) that the condition band and the valence band are touched may be written as $H = v_F \vec{p}.\vec{\sigma}$ where $\vec{p} = \hbar \vec{k}$ is the momentum of the particles and σ are Pauli spin matrices

acting on the honeycomb sub-lattice degrees of freedom [3]. This is the Dirac equation for massless relativistic particles with linear energy spectrum in 2D. Many of the interesting electronic properties in graphene are resulted from this dispersion relation [5].

When electrons are confined in two-dimensional (2D) systems, at low temperatures and inserted strong magnetic field perpendicular, the quantum Hall effects (QHE) can be observed that is a manifestation of quantum mechanically enhanced transport phenomena on the Landau electronic levels. The early indication for the quantum effects on transport phenomena of the 2D electrons inside a strong magnetic field was found by Shubnikov and De Haas in 1930.

They found that above a threshold field, the longitudinal resistivity oscillates as a function of the field [2, 6]. In the QHE, the Hall resistance is $\sigma_{xy} = \nu \frac{e^2}{h}$, where ν is filling factors and could be an integer or rationale number respectively.However, the quantization condition in graphene is distinctively different and in particular, very unusual half-integer quantum Hall effects were discovered. It provides an unambiguous evidence of the existence of Dirac fermions so graphene is strongly peculiar from the other conventional 2D electronic systems (containing carriers with finite mass) [1, 4].

Moreover, the quantization of dynamic systems in classical phase space were implemented by Weyl –Wigner –Groenewold – Moyal method. The WWGM method works well for observables possessing a classical limit. The method shows in a non-commutative space, the coordinate operators satisfies the following commutation relation

$$\left[x^{\mu}, x^{\nu} \right] = i\,\theta^{\mu\nu}$$

Where $\theta^{\mu\nu}$ is the anti-symmetric matric and the ordinary products are replaced by the star ones [7, 8]:

$$*= exp\left[\frac{i\hbar}{2}\left(\frac{\overleftarrow{\partial}}{\partial x^{\mu}}\frac{\overrightarrow{\partial}}{\partial p_{\mu}} - \frac{\overleftarrow{\partial}}{\partial p^{\mu}}\frac{\overrightarrow{\partial}}{\partial x_{\mu}} \right) \right]$$

Corresponding to the quantum commutators one considers the Moyal bracket of two observables $f(P,Q)$ and $g(P,Q)$ is:

$$[f(P,Q), g(P,Q)] = f(P,Q) * g(P,Q) - g(P,Q) * f(P,Q)$$

In addition the Moyal bracket of the matrices $M_{ab}(P,Q)$ and $N_{ab}(P,Q)$ can be defined as:

$$([M(P,Q), N(P,Q)]_*)_{ab} = M_{ac}(P,Q) * N_{cb}(P,Q) - N_{ac}(P,Q) * M_{cb}(P,Q)$$

To take into account this fact we define the "semi classical" bracket

$$\{M(P,Q), N(P,Q)\}_C = \frac{-i}{\hbar}[M,N] + \frac{1}{2}\{M(P,Q), N(P,Q)\} - \frac{1}{2}\{M(P,Q), N(P,Q)\}$$

Where the first term on the right hand side is the ordinary commutator of the matrices and the last two terms are Poisson brackets [9].

The inserted constraints are specified in classification of the gauge theories and in this regard, a quantity such that its Poisson brackets with all the constraints are weakly vanish, is first class constraint (FCC) otherwise it is second class constraint (SCC). Systems that have FCC, called gauge theory [10].

Appearance of SCC, breaks the gauge symmetry, and for quantization of the SCC systems we must use Dirac brackets instead of Poisson brackets. However this procedure may implies some difficulties such as factor ordering. The BFT approach introduced by Batalin, Fradkin and Batalin – Tyutin gives an alternative

method for quantization of the SCC systems [11, 12].

This method is based on extending the phase space to include a set of new variables and then writing the constraints, similar to the physical quantities, as a power series in terms of these added variables [13-15].

Here, we investigate moving of an electron in a 2D graphene system as a SCC system. So according to the BFT method, we convert the SCC system to the FCC and introducing a new gauged system.

In section 2 we use the Lagrangian and structure of constraints for the graphene systems in the non-commutative space. In section 3 we study moving of an electron in 2D space then in section 4 we briefly review BFT method. In section 5 we apply the BFT approach to the graphene system and finally the section 6 is devoted to conclusion.

2. Graphene Systems in Non-commutative Space

We consider a 2D non-commutative space, the first order matrix Lagrangian adequate to formulate spin dynamics in this space, is [7, 9]

$$L = \dot{r}^\alpha \left[\frac{p_\alpha}{2} I + \rho A_\alpha(r) \right] - \frac{\dot{p}^\alpha}{2} I \left[r_\alpha + \frac{\theta_{\alpha\beta}}{\hbar} p_\beta \right] \tag{1}$$
$$- H_0(r,p)$$

I denotes the unit matrix possessing the same dimension with the matrix valued gauge field A_α. We denoted the coupling constants ρ. Although (1) is classical, $\hbar$ is present to furnish the constant and anti-symmetric non-commutativity parameter $\theta_{\alpha\beta}$ with the dimension (length)^2. The canonical

momenta $P_I = \left(P_r^{\;\alpha} = \frac{\partial L}{\partial \dot{r}_\alpha}, P_p^{\;\alpha} = \frac{\partial L}{\partial \dot{p}_\alpha} \right)$ corresponding to the coordinates $Q_I = (r_\alpha, p_\alpha)$ yield the dynamical constraints. [8]

$$\psi^{1\alpha} = \left(\vec{P}_r^{\;\alpha} - \frac{1}{2} p^{\;\alpha} \right) I - \rho \vec{A}^{\;\alpha} \tag{2}$$

$$\psi^{2\alpha} = \left(\vec{P}_p^{\;\alpha} + \frac{1}{2} r^{\;\alpha} \right) I + \frac{\theta^{\alpha\beta}}{\hbar} p_\beta \tag{3}$$

They satisfy the semi classical relations

$$\left\{ \psi^{1\alpha}, \psi^{1\beta} \right\}_C = \rho F_{\alpha\beta} \tag{4}$$

$$\left\{ \psi^{2\alpha}, \psi^{2\beta} \right\}_C = \frac{\theta_{\alpha\beta}}{\hbar} \tag{5}$$

$$\left\{ \psi^{1\alpha}, \psi^{2\beta} \right\}_C = -\delta_{\alpha\beta} \tag{6}$$

$$F_{\alpha\beta} = \frac{\partial A_\beta}{\partial r_\alpha} - \frac{\partial A_\alpha}{\partial r_\beta} - \frac{i\rho}{\hbar} \left[A_\alpha, A_\beta \right] \tag{7}$$

Where $F_{\alpha\beta}$ is the field strength and $\delta_{\alpha\beta}$ is the Keroneker delta. We can introducing the semi-classical Dirac brackets

$$\left\{ M, N \right\}_{CD} =$$
$$\left\{ M, N \right\}_C - \left\{ M, \psi^z \right\}_C C_{zz'}^{-1} \left\{ \psi_{z'}, N \right\} \tag{8}$$

Where C^{-1} is the inverse of $C_{\alpha\beta}^{zz'} = \left\{ \psi_\alpha^z, \psi_\beta^{z'} \right\}_C$.

We can extend the canonical quantization rules to the matrix observation by $\{ , \}_{CD} \rightarrow \frac{1}{i\hbar}[,]$, and with distinguish the matrix commutators and relations (4-6) and quantum commutation relations, can introducing generalized algebra:

$$\left\lceil \hat{r}^\alpha, \hat{r}^\beta \right\rceil_q = i\theta^{\alpha\beta} \tag{9}$$

$$\left\lceil \hat{p}^\alpha, \hat{p}^\beta \right\rceil_q = i\hbar\rho F^{\alpha\beta} - i\rho^2 (F\theta F)^{\alpha\beta} \tag{10}$$

$$\left\lceil \hat{r}^\alpha, \hat{p}^\beta \right\rceil = i\hbar\delta^{\alpha\beta} - i\rho(\theta F)^{\alpha\beta} \tag{11}$$

Note that we keep first order θ and if $\theta = 0$, then relations (9-11) come back commutative

space. A realization of the generalized algebra (9–11) and a Hamiltonian $H_0(p,r)$ should be provided.

$$\hat{p}^{\alpha} = D_{\alpha} - \frac{\rho}{2\hbar} F_{\alpha\beta}\theta^{\beta\gamma} D_{\gamma} \qquad (12)$$

$$\hat{r}^{\alpha} = r_{\alpha} - \frac{1}{2\hbar}\theta_{\alpha\beta} D^{\beta} \qquad (13)$$

$$D_{\alpha} = -i\hbar\frac{\partial}{\partial r^{\alpha}} - \rho A_{\alpha} \qquad (14)$$

(12-14) satisfy the algebra (9-11). To illustrate the method let the initial Hamiltonianbe $H_0(p) = \frac{p^2}{2m}$. Substituting p with the quantum operator (12) one obtains the θ–deformed Hamiltonian

$$H_0(\hat{p}) \equiv \hat{H}_{nc} = \frac{1}{2m}\left(D_{\alpha} - \frac{\rho}{2\hbar}F_{\alpha\beta}\theta^{\beta\gamma}D_{\gamma}\right)^2 \qquad (15)$$

3. Electron on 2D plane

Now, if we consider an electron moving on the two-dimensional plane $r_i = (x,y)$ in graphene in the presence of the uniform external in-plane electric field **E** and the uniform external perpendicular magnetic field **B**, and the field strength $F_{ij} = \varepsilon_{ij}B$ and constant $\rho = -\frac{e}{c}$.

$$\lceil \hat{x},\hat{y} \rceil_q = i\theta \qquad (16)$$

$$\lceil \hat{p}_i,\hat{p}_j \rceil_q = -\frac{ieB\theta}{c}\left(1 - \frac{eB\theta}{\hbar c}\right)\varepsilon_{ij} \qquad (17)$$

$$\lceil \hat{r}_i,\hat{p}_j \rceil_q = i\hbar\left(1 - \frac{eB\theta}{\hbar c}\right)\delta_{ij} \qquad (18)$$

Choosing the electric field to lie in the direction of the x-axis, and by choosing symmetric gauge $A_i = \frac{eB}{2c}\varepsilon_{ij}r_j$, the Hamiltonian in non-commutative space is taken as:

$$\bar{H}_{nc} = \frac{1}{2}\hat{p}_i^2 + eE\hat{x}$$

$$= \frac{1}{2m}\left(D_{\alpha} - \frac{\rho}{2\hbar}F_{\alpha\beta}\theta^{\beta\gamma}D_{\gamma}\right)^2 + eE\hat{x} \qquad (19)$$

$$\hat{p}_i^{(1)} = \left(1 - \frac{eB\theta}{2\hbar c}\right)p_i - \frac{eB}{2c}\varepsilon_{ij}r^j \qquad (20)$$

$$\hat{r}_i^{(1)} = \left(1 - \frac{eB\theta}{4\hbar c}\right)r_i - \frac{\theta}{2c}\varepsilon_{ij}p^j \qquad (21)$$

By plugging (20) and (21) into (19) one obtains the Hamiltonian [8,9].

$$H_{nc} = \frac{1}{2m}\left[(1 - 2\kappa)p_i - \frac{eB}{2c}\varepsilon_{ij}r^j\right]^2 + eE(1-\kappa)x - \frac{eE\theta}{2\hbar}p_y \qquad (22)$$

Where we defined $p_i = -i\hbar\nabla_i$ and $\kappa = \frac{eB\theta}{4\hbar c}$.

4. Review of the BFT Method

We consider a SCC system described by Hamiltonian H_0 in phase space with coordinate (q^i, p_j) that $i = 1,2,...,k$. Assume we are a given a set of SCC Ω_{α}, $\alpha = 1,2,...,m$ satisfying the algebra:

$$\Delta_{\alpha\beta} = \left\{\Omega_{\alpha},\Omega_{\beta}\right\} \qquad (23)$$

Where $\{\ ,\ \}$ is the Poisson bracket and $\Delta_{\alpha\beta}$ is an invariant matrix. To convert this second class system into a gauge system, i.e. a first class system, one should extend the phase space by introducing the same number of auxiliary variables as that of SCC. We denote variables by η^{α} with following algebra:

$$\left\{\eta^{\alpha},\eta^{\beta}\right\} = \omega^{\alpha\beta} \qquad (24)$$

Where $\omega^{\alpha\beta}$ is an anti-symmetric matrix which we assume it to be constant. The FCC in the extended phase space $(q,p)\oplus(\eta)$ are defined as

$$\tau_{\alpha} = \tau_{\alpha}(q,p,\eta) \qquad \alpha = 1,2,...,m \qquad (25)$$

With the boundary conditions

$$\tau_{\alpha}(q,p,0) = \Omega_{\alpha}(q,p) \qquad (26)$$

In the Abelian BFT embedding method one demands that these extended constraints are strongly involutes:

$$\{\tau_\alpha, \tau_\beta\} = 0 \tag{27}$$

The solution of the above equation can be obtained by considering τ_α as:

$$\tau_\alpha = \sum_{n=0}^{\infty} \tau_\alpha^{(n)} \tag{28}$$

Where $\tau_\alpha^{(n)}$ is order n with respect to η^α s. according to the boundary condition (26) we have

$$\tau_\alpha^{(0)} = \Omega_\alpha \tag{29}$$

Substituting (28) into (27), lead to a set of recursive relation. Vanishing the term independent of η gives:

$$\{\tau_\alpha^{(0)}, \tau_\beta^{(0)}\} + \{\tau_\alpha^{(1)}, \tau_\beta^{(1)}\}_{(\eta)} = 0 \tag{30}$$

And vanishing the term of order n with respect to η^α's for $n \geq 1$ gives

$$\{\tau_{[\alpha}^{(1)}, \tau_{\beta]}^{(n+1)}\}_{(\eta)} + B_{\alpha\beta}^{(n)} = 0 \qquad n \geq 1 \tag{31}$$

Where:

$$B_{\alpha\beta}^{(n)} \equiv \{\tau_{[\alpha}^{(0)}, \tau_{\beta]}^{(1)}\} \tag{32}$$

$$B_{\alpha\beta}^{(n)} \equiv \frac{1}{2} B_{[\alpha\beta]} \equiv \sum_{m=0}^{n} \{\tau_\alpha^{(n-m)}, \tau_\beta^{(m)}\}$$
$$+ \sum_{m=0}^{n-2} \{\tau_\alpha^{(n-m)}, \tau_\beta^{(m+2)}\}_{(\eta)} \qquad n \geq 2 \tag{33}$$

The suffix η in the above equation means that the Poisson brackets must be evaluated with respect to η variable only, otherwise they are calculated in the basic (q, p). The above equation are used iteratively to obtain the correction terms $\tau^{(n)}$. Since $\tau^{(1)}$ is linear with respect to η we may write:

$$\tau_\alpha^{(1)} = \chi_{\alpha\beta}(q, p)\eta^\beta \tag{34}$$

Substituting this expression into (30) and using (23) and (24) we obtain:

$$\Delta_{\alpha\beta} + \chi_{\alpha\gamma}\omega^{\gamma\lambda}\chi_{\beta\lambda} = 0 \tag{35}$$

This equation contains two unknown elements, $\chi_{\alpha\beta}$ and $\omega^{\alpha\beta}$. One should at first assume a suitable anti-symmetric matrix for $\omega^{\alpha\beta}$ and then solve (35) to determine the coefficient equation for $\chi_{\alpha\beta}$. Since $\Delta_{\alpha\beta}$ and $\omega^{\alpha\beta}$ are anti-symmetric matrices, there are totally $\frac{m(m-1)}{2}$ independent equation for $\chi_{\alpha\beta}$, while the number of $\chi_{\alpha\beta}$'s are m^2. Therefore there existan infinite number of solutions for $\chi_{\alpha\beta}$ and we are allowed to choose any solution we wish. Using this possibility, $\chi_{\alpha\beta}$'s can be chosen such that the process of determining the correction terms $\tau_\alpha^{(n)}$ terminate at this stage, i.e. τ^2 vanishes. We will come to this point in the next section. It can be seen that the general solution of (31) is given by [16].

$$\tau_\alpha^{(n+1)} = -\frac{1}{n+2}\eta^\mu \omega_{\mu\nu}\chi^{\nu\rho}B_{\rho\alpha}^{(n)} \qquad n \geq 1 \tag{36}$$

To construct corresponding Hamiltonian $\tilde{H}(p, q, \eta)$ in the extended phase space we demand

$$\tilde{H} = \sum_{n=0}^{\infty} \tilde{H}^{(n)} \tag{37}$$

Where $\tilde{H}^{(n)}$is of order n with respect to η^α's. Such that

$$H(q, p, 0) = H_0(q, p) \tag{38}$$
$$\{\tau_\alpha, \tilde{H}\} = 0$$

Substituting from (28) and (37) in (38) gives:

$$\{\tau_\alpha^{(1)}, \tilde{H}^{(n+1)}\}_\eta + G_\alpha^{(n)} = 0 \qquad n \geq 0 \tag{39}$$

Where $G_\alpha^{(n)}$ as the generators of the $\tilde{H}^{(n+1)}$ are defined as the following

$$G_\alpha^{(0)} \equiv \left\{\tau_\alpha^{(0)}, H^{(0)}\right\} \tag{40}$$

$$
\begin{aligned}
G_\alpha^{(1)} &\equiv \left\{\tau_\alpha^{(1)}, \tilde{H}^{(0)}\right\} + \left\{\tau_\alpha^{(0)}, \tilde{H}^{(1)}\right\} \\
&+ \left\{\tau_\alpha^{(2)}, \tilde{H}^{(1)}\right\}_\eta
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
G_\alpha^{(n)} &\equiv \sum_{m=0}^{n} \left\{\tau_\alpha^{(n-m)}, \tilde{H}^{(m)}\right\} \\
&+ \sum_{m=0}^{n-2} \left\{\tau_\alpha^{(n-m)}, \tilde{H}^{(m+2)}\right\}_{(\eta)} \\
&+ \left\{\tau_\alpha^{(n+1)}, \tilde{H}^{(1)}\right\}_{(\eta)} \qquad n \geq 2
\end{aligned}
\tag{42}
$$

It can be shown that the general expression for $\tilde{H}^{(n)}$ is:

$$\tilde{H}^{(n+1)} = -\frac{1}{n+1}\eta^\alpha \omega_{\alpha\beta} \chi^{\beta\nu} G_\nu^{(n)} \tag{43}$$

This completes the BFT method of converting a second class system to a strongly involution first class one. As can be seen the correction terms $\tau_\alpha^{(n)}$ and $\tilde{H}^{(n)}$ are derived iteratively from (36) and (43). Generally, there is no guarantee that the series terminate at some definite order. However, the series will terminate if $B_{\alpha\beta}^{(n)}$ and $G_\alpha^{(n)}$ vanish for a certain order $n = N$ [13-15].

5. Embedding Method on a Graphene Model

In this section we have preform BFT at an electron that moving on 2D plan in graphene(see section 3). By consider SCC s (2) and (3) and Hamiltonian (22) we use these possibilities to find a systematic method to truncate infinite series encountered in BFT method. We should

solve the iterative equation for $\tau_\alpha^{(n)}$ and $\tilde{H}^{(n)}$. By using (4), one can write Δ-matrix (23) as

$$
\Delta = \begin{pmatrix}
0 & \dfrac{eB}{c} & -1 & 0 \\[2mm]
-\dfrac{eB}{c} & 0 & 0 & -1 \\[2mm]
1 & 0 & 0 & \dfrac{\theta}{\hbar} \\[2mm]
0 & 1 & -\dfrac{\theta}{\hbar} & 0
\end{pmatrix}
\tag{44}
$$

We introduce SCC in non-commutative space as:

$$
\begin{aligned}
\left\{\psi^{1\alpha}, \psi^{1\beta}\right\}_C &= \rho F_{\alpha\beta} \\
\left\{\psi^{2\alpha}, \psi^{2\beta}\right\}_C &= \frac{\theta_{\alpha\beta}}{\hbar} \\
\left\{\psi^{1\alpha}, \psi^{2\beta}\right\}_C &= -\delta_{\alpha\beta}
\end{aligned}
\tag{45}
$$

Where ρ is constant, $F_{\alpha\beta} = \varepsilon_{\alpha\beta}B$, and $\delta_{\alpha\beta}$ is keroneker delta and $\theta_{\alpha\beta}$ is constant and anti-symmetric matrix .We consider

$$\Psi_1 \equiv \psi^{11} \qquad \Psi_3 \equiv \psi^{21}$$
$$\Psi_2 \equiv \psi^{12} \qquad \Psi_4 \equiv \psi^{22} \tag{46}$$

In this case Δ is constant, and it is easily that choice $\omega = -\Delta$ and $\chi = 1$, then solve the basic equation (35) .We know that $\tau_\alpha^{(0)}$, thus posses

$$\tau_1^{(0)} = \Psi_1 \qquad \tau_3^{(0)} = \Psi_3$$
$$\tau_2^{(0)} = \Psi_2 \qquad \tau_4^{(0)} = \Psi_4 \tag{47}$$

By this choice we write as: $\tau_\alpha^{(1)} = \chi_{\alpha\beta}\eta^\beta$, and we have

$$\tau_1^{(1)} = \eta^1 \qquad \tau_3^{(1)} = \eta^3$$
$$\tau_2^{(1)} = \eta^2 \qquad \tau_4^{(1)} = \eta^4 \tag{48}$$

Since $\tau_\alpha^{(1)}$ is only a function of η, it can be seen for $n \geq 1$, all other $B_{\alpha\beta}^{(n)}$ vanish, thus $\tau^\alpha = \tau_\alpha^{(n)} + \eta^\alpha$ and one can write as:

$$\tau^1 = \Psi_1 + \eta^1$$
$$\tau^2 = \Psi_2 + \eta^2$$
$$\tau^3 = \Psi_3 + \eta^3 \quad (49)$$
$$\tau^4 = \Psi_4 + \eta^4$$

That leads to the following finite order embedding for the constraints.

Now, for complete our procedure we should also construct the extended Hamiltonian. Inserting ω, Δ and χ into (43), the correction terms of Hamiltonian as:

For $n = 0$ then

$$\tilde{H}^{(1)} = G_1^{(0)}\left[\eta^3 - \frac{eB}{c}\eta^2\right] + G_2^{(0)}\left[\frac{eB}{c}\eta^1 + \eta^4\right]$$
$$- G_3^{(0)}\left[\eta^1 + \frac{\theta}{\hbar}\eta^4\right] + G_4^{(0)}\left[\frac{\theta}{\hbar}\eta^3 - \eta^2\right] \quad (50)$$

$G_\alpha^{(0)}$ Can be derived from (40), and give:

$$G_1^{(0)} = -eE(1-\kappa) \quad (51)$$

$$G_2^{(0)} = 0 \quad (52)$$

$$G_3^{(0)} = \left[\frac{(1-2\kappa)}{m}\right]\left[(1-2\kappa)p_\alpha - A_\alpha\right] \quad (53)$$

$$G_4^{(0)} = \left[\frac{(1-2\kappa)}{m}\right]\left[(1-2\kappa)p_\alpha - A_\alpha\right] - \frac{eE\theta}{2\hbar} \quad (54)$$

For $n = 1$ then

$$\tilde{H}^{(2)} = \frac{1}{2}G_1^{(1)}\left[\eta^3 - \frac{eB}{c}\eta^2\right] + \frac{1}{2}G_2^{(1)}\left[\frac{eB}{c}\eta^1 + \eta^4\right]$$
$$- \frac{1}{2}G_3^{(1)}\left[\eta^1 + \frac{\theta}{\hbar}\eta^4\right] + \frac{1}{2}G_4^{(0)}\left[\frac{\theta}{\hbar}\eta^3 - \eta^2\right] \quad (55)$$

$G_\alpha^{(1)}$ Can be derived from (41), and give:

$$G_1^{(1)} = \frac{eB}{2c}\left[\frac{(1-2\kappa)}{m}\right]\left[\frac{\theta}{\hbar}\eta^3 - \eta^2\right] \quad (56)$$

$$G_2^{(1)} = -\frac{eB}{2c}\left[\frac{(1-2\kappa)}{m}\right]\left[\eta^1 + \frac{\theta}{\hbar}\eta^4\right] \quad (57)$$

$$G_3^{(1)} = -\left[\frac{(1-2\kappa)^2}{m}\right]\left[\eta^1 + \frac{\theta}{\hbar}\eta^4\right] \quad (58)$$

$$G_4^{(1)} = \left[\frac{(1-2\kappa)^2}{m}\right]\left[\frac{\theta}{\hbar}\eta^3 - \eta^2\right] \quad (59)$$

For $n \geq 2$, $G_\alpha^{(n)}$ vanish, thus consequently $\tilde{H}^{(3)} = 0$.

In this case one can finally write:

$$H = H^{nc} + \tilde{H}^{(1)} + \tilde{H}^{(2)} \quad (60)$$

Equations (49) and (60) represent a finite order gauge theory in BFT method.

6. Conclusion

We consider a SCC system described by Hamiltonian H_{nc} for a 2D graphene in non-commutative space. We know SCC illustrate break of gauge symmetry, where Dirac quantization encountering with some problems.

In order to remove the problems, we use BFT approach which by extension of the phase space and introducing a series of auxiliary fields the SCC converts the FCC so the ordinary direct gauge quantization methods would be applicable. The series of constrains and the extended Hamiltonian would have infinite terms but by choosing a set of suitable parameters ($\Delta_{\alpha\beta}$ and $\omega^{\alpha\beta}$) it is possible to achieve a finite order of the BFT method and hence to make the constrains conversion. In

continue, the extended new Hamiltonian could be gauge quantized routinely.

Our work describes the graphene in a 2D non-commutative plane as a fully gauged symmetrical model.

Acknowledgments

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