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Surface Effect on Vibration of Y-SWCNTs Embedded on Pasternak Foundation Conveying Viscose Fluid

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Abstract

Surface and small scale effects on free transverse vibration of a single-walled carbon nanotube (SWCNT) fitted with Y-junction at downstream end conveying viscose fluid is investigated in this article based on Euler-Bernoulli beam (EBB) model. Nonlocal elasticity theory is employed to consider small scale effects due to its simplicity and efficiency. The energy method and Hamilton's principle are used to establish the corresponding motion equation. To discretize and solve the governing equation of motion the Galerkin method is applied. Moreover, the small-size effect, angle of Y-junction, surface layer and Pasternak elastic foundation are studied in detail. Regarding fluid flow effects, it has been concluded that the fluid flow is an effective factor on increasing the instability of Y-SWCNT. Results show that increasing the angle of Y-junction enhances the flutter fluid velocity where the first and second modes are merged. This work could be used in medical application and design of nano-electromechanical devices such as measuring the density of blood flowing through such nanotubes.

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1. Introduction

After the discovery of carbon nanotubes (CNTs) by Iijima [1] many experimental and theoretical investigations of CNT-based multi terminal structures, namely, junctions of L, X, Y, and T-types, have been carried out [2-4]. It has been shown that such structures can be used as nanotransistors and nanodiodes. Y junctions were fabricated with a stem exceeding the branches in diameter and with an acute angle between them. A Y-shaped single-wall carbon nanotube(Y-SWCNTs) is a novel structure consisting of three terminals with different chirality [5]. Y-SWCNTs have a potential to realize the nanoscale three-terminal devices where the third terminal could be used for controlling the switching, power gain, or other applications [6, 7]. Results showed that the junctions have high thermal stability and mechanical strength which made CNT junctions as one of the most important devices to improve the future of nanoscale structures [8]. Therefore In recent years the syntheses of CNTs and carbon nanorods (CNRs) with Y or T branched junctions have become a focus in carbon material research because of their potential ability to bring novel mechanical, electrical, thermal properties to nanodevices [9]. Future investigations should correlate the detailed physical structure of the nanostructure, especially the study of Y-junctions which is still in its infancy.

In this study, the nonlocal elasticity theory is used which was first introduced and developed by [10-12] to consider small scale effect in the continuum model of nanostructures. In recent years, studies about the vibration of nanostructures using the nonlocal theory of elasticity are increased due to superior vibration characteristics of them. Based on the partial nonlocal elasticity theory, Ghorbanpour Arani et al. [13, 14] investigated the free transverse vibrations of SWCNT and double-walled carbon nano-tube (DWCNT) under axial load using the Euler-Bernoulli beam (EBB), Timoshenko beam (TB) and Donnell shell models. An elastic rod model was developed by Chang, [15] to study the small scale effect on axial vibration of non-uniform and non-homogeneous nanorods by using the theory of nonlocal elasticity.

Since nanotubes conveying fluid can be used in wide range of modern engineering structures; recently, a large amount of research works have been carried out on the buckling and vibration of the nanotubes conveying fluid. Among these studies, beam and shell models are commonly utilized to consider the effects of fluid flow on the nanotubes. Kuang et al. [16] analyzed nonlinear vibrations of DWCNTs conveying fluid. They considered both uncoupling and coupling between the longitudinal and transverse displacements. Vibration and instability analysis of CNTs conveying fluid was investigated by Ghavanloo et al. [17]. They reported that the effect of internal moving fluid is characterized by two parameters, the steady flow velocity and the mass density of the fluid. In some recent literatures the internal fluid flow is assumed to be viscous.

Since at nanoscale problems surface-to-bulk energy ratio increases, surface effects must be taken into account, while it can be disregarded in macroscopic structural problems. The effects of surface residual stresses on nano-beams were studied by Bar On and Altus [18]. The nonlinear equations were separated into two complementary parts; static, which includes the surface residual moments and yields a residual deflection, and dynamic, for the beam vibrations associated with the residual deflection via geometrical nonlinearity. Lee and Chang [19] investigated surface and small-scale effects on vibration analysis of a non-uniform nano-cantilever beam. They found that the surface effects with positive surface constants tend to increase the critical axial force and the natural frequency and shear deformation tends to decrease the critical axial compression force and the natural frequency.

2. Formulation

Fig. 1 illustrates a schematic diagram of a Y-SWCNT conveying fluid with inner radius R_i, outer radius R_o, thickness h, length L and the angle of Y-junction ϕ embedded in a Pasternak elastic medium with considering surface layer at outer surface of CNT that covered with crystal of nickel.

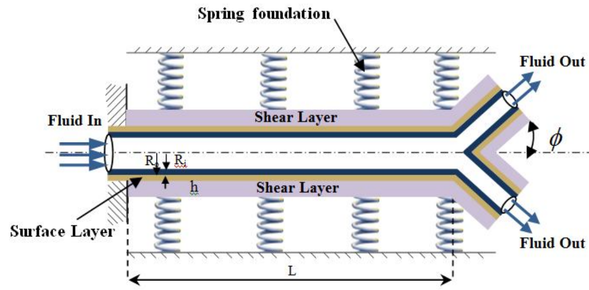

Fig. 1. Configuration of embedded Y-SWCNT conveying fluid

2.1. Surface Effect

For appropriate modeling of the Y-SWCNT an additional bonded thin surface layer (outer layer) is encompassed the Y-SWCNT. Since at nanoscale problems the ratio of surface-to-volume becomes significant, the effects of surface layer cannot be ignored. Materials properties often used in surface science are surface stress and surface energy. The reversible work per unit area needed to elastically enlarge a pre-existing surface called surface stress. Gurtin and Murdoch [20] presented a mathematical frame work to study the mechanical behavior of material surfaces. The other material property is surface energy which is defined as the reversible work per unit area concerned in forming a surface. For the surface of elastic material the following basic relation can be written as:

$$\tau_{\alpha\beta} = \gamma\delta_{\alpha\beta} + \frac{\partial \Gamma}{\partial \varepsilon_{\alpha\beta}}, \quad (\alpha, \beta = 1,2), \tag{1}$$

where $\tau_{\alpha\beta}$ is the surface stress tensor, γ is surface residual energy in the absence of external loading and surface deformation and Γ denotes the induced surface energy. Applying the Hook's law in the Eq. (1) yields the following stress-displacement relation as [20]:

$$\tau_{\alpha\beta} = \tau\delta_{ij} + \lambda^S \varepsilon_{\eta\eta}\delta_{\alpha\beta} + 2\mu^S \varepsilon_{\alpha\beta}, (i, j = 1,2,3), \tag{2}$$

in which λ^s and μ^s are the Lame constants of surface material, and τ denotes the surface residual stress. The generalized Young-Laplace equation can be expressed in the cases with zero thickness as follows:

$$\Delta\sigma_{ij} n_i n_j = \tau_{\alpha\beta}\kappa_{\alpha\beta}, \tag{3}$$

where $\Delta\sigma_{ij}$ is the stress jump across each external interface surface, n_i is the unit normal vector and $\kappa_{\alpha\beta}$ is the curvature tensor.

It is assumed that the thickness of the surface layer h^S is much smaller than the thickness h and inner radius R_i of Y-SWCNT and also, the thickness of the thin surface layer approaches zero; hence, the Laplace-Young equations (Wang [21]) can be used to express the surface residual stress as:

$$\tau_{xz} = 4\tau_0 R_o \frac{\partial^2 w}{\partial x^2}, \tag{4}$$

in which τ_0 is the residual surface tension. Also, the work done by the resulting distributed transverse loading induced by surface effects can be written as follow:

$$q_s = \frac{1}{2}\int_0^L 4\tau_0 R_0 \frac{\partial^2 w}{\partial x^2} w dx. \tag{5}$$

Considering the additional flexural rigidity due to outer surface layer and using the composite beam theory [21] yields the effective flexural rigidity as:

$$(EI_t)^* = EI_t + E^S \pi h^S R_o^3, \tag{6}$$

where E^S is the Young's modulus of surface layer.

2.2. Foundation Effect

The effects of the Pasternak surrounding elastic medium on the Y-SWCNT with a Y-junction fitted at the downstream are considered as follows:

$$q_e = \frac{1}{2}\int_0^L (-k_w w + G\nabla^2 w) w dx, \tag{7}$$

in which k_w and G are the spring constant of the Winkler type, the shear constant of the Pasternak type, respectively.

2.3. Strain and Kinetic Energies

Total potential energy V and total kinetic energy T of Y-SWCNT associated with the vibration of CNT, fluid flow at CNT and fluid flow at the downstream Y-junction defined as:

$$V = \frac{1}{2}\int_0^L \int_{A_t} (\sigma_{xx}\varepsilon_{xx}) dx dA_t, \tag{8}$$

$$T = \frac{1}{2}\int_0^L \left[m_t + m_e\delta(x-L)]v_t^2 + m_f v_f^2 + m_{fe} v_{fe}^2 \delta(x-L) \right] dx, \tag{9}$$

Here m_t, m_f, m_e, m_{fe}, v_t, v_f, v_{fe} and δ are the mass per unit length of SWCNT, the mass per unit length of fluid, the mass of downstream elbows modeled as point mass, the mass of the downstream flow fluid at the elbows modeled as point mass, perturbation velocity of SWCNT, velocity of fluid, velocity of downstream flow fluid at the elbows and delta Dirac function, respectively.

2.4. Viscosity and Y-junction Effect

The well-known Navier-Stokes equation can be used to consider the effect of viscosity of fluid flow as [22]:

$$\rho_f \frac{dv_f}{dt} = -\nabla p + \mu \nabla^2 v_f , \tag{10}$$

in which p is pressure and μ is the viscosity of the fluid. The viscosity effect derived from Eq. (10) as follow:

$$q_\mu = \frac{1}{2}\int_0^L A_f \mu \frac{\partial^2 v_f}{\partial x^2} dx. \tag{11}$$

Also the work done by fluid flow at the Y-junction due to the momentum equation for incoming and out coming of fluid at Y-junction can be written as:

$$q_{Yj} = \frac{1}{2}\int_0^L m_f v_f^2 (1-\cos\phi)\frac{\partial^2 w}{\partial x^2} w dx. \tag{12}$$

2.5. Motion equation

To derive the motion equation of embedded Y-SWCNT conveying viscose fluid, the Hamilton's principle is employed, as follow:

$$\delta \int_0^t (V - T - q_s - q_e - q_\mu - q_{Yj}) dt = 0. \tag{13}$$

Substituting Eq. (5), (7-9) and (11-12) into Eq. (13) and setting the coefficients of δw to zero, yields the following differential equation of motion as:

$$\begin{aligned}
&-\frac{\partial^2 M_x}{\partial x^2} + (k_w w - G\nabla^2 w + m_f v_f^2 \cos\phi - m_f v_f^2 (1-\cos\phi) \\
&-4\tau_0 R_o)\frac{\partial^2 w}{\partial x^2} - \mu A_f v_f \frac{\partial^3 w}{\partial x^3} - \mu A \frac{\partial^3 w}{\partial x^2 \partial t} + \\
&2v_f (m_f + m_{fe}\delta(x-L))\frac{\partial^2 w}{\partial x \partial t} - (\rho_t I_t + \rho_f I_f)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \\
&(m_t + m_f + m_{fe}\delta(x-L) + m_e\delta(x-L))\frac{\partial^2 w}{\partial t^2} = 0
\end{aligned} \tag{14}$$

2.6. Nonlocal Beam Model

The constitutive equations of the partial nonlocal elasticity can be written as:

$$\left[1-(e_0 a)^2 \nabla^2\right]\sigma^{nl} = \sigma^l , \tag{15}$$

where $e_0 a$ is the small scale parameter, σ^{nl} is the nonlocal stress tensor and σ^l denotes the classical stress, respectively.

Also base on nonlocal elasticity the moment resultant M_x can be expressed as:

$$M_x - (e_0 a)^2 \frac{\partial^2 M_x}{\partial x^2} = -(EI_t)^* \frac{\partial^2 w}{\partial x^2}. \tag{16}$$

Substituting Eq. (16) into Eq. (14) and introduce the following non-dimensional parameters, the equation of motion for transverse vibration is derived:

$$\begin{aligned}
&\overline{x} = \frac{x}{L}, \overline{w} = \frac{w}{L}, \overline{t} = \sqrt{\frac{EI_t}{m_f + m_t}}\frac{t}{L^2}, \overline{G} = \frac{G}{A_t E}, \overline{K}_w = \frac{K_w L^4}{EI_t}, \\
&\overline{\xi} = \frac{A_t L^2}{I_t}, \overline{v}_f = \sqrt{\frac{m_f}{EI_t}} v_f L, \beta = \frac{m_f}{m_f + m_t}, \overline{h} = \frac{E^s \pi h^s R_o^3}{EI_t}, \\
&\overline{\beta}_f = \frac{m_{fe}}{L\sqrt{m_f^2 + m_f m_t}}, \overline{\beta}_{fe} = \frac{m_{fe} + m_e}{L(m_f + m_t)}, \overline{\mu} = \frac{\mu A_f}{\sqrt{EI_t m_f}}, \\
&\overline{\sigma} = \frac{\rho_t I_t + \rho_f I_f}{L^2 (m_f + m_t)}, \overline{e}_n = \frac{e_0 a}{L}, \overline{\tau} = \frac{4\tau R_0}{A_t E}, \overline{\varepsilon} = \frac{2E^s h^s}{E(R_0^2 - R_i^2)}.
\end{aligned} \tag{17}$$

Substituting Eq. (16) into Eq. (14) and using dimensionless parameters mentioned in Eq. (17) the motion equation for transverse vibration can be derived as:

$$\begin{aligned}
&\bar{\sigma}\bar{e}_n^2\frac{\partial^6\bar{w}}{\partial\bar{t}^2\partial\bar{x}^4}+\left[\bar{g}\left(1+\bar{h}\right)-\bar{e}_n^2\bar{\mu}\sqrt{\bar{\beta}}\right]\frac{\partial^5\bar{w}}{\partial\bar{x}^4\partial\bar{t}}\\
&-\bar{e}_n^2\bar{\mu}\bar{v}_f\frac{\partial^5\bar{w}}{\partial\bar{x}^5}-2\bar{e}_n^2\bar{v}_f\left[\sqrt{\bar{\beta}}+\bar{\beta}_f\delta\left(\bar{x}-1\right)\right]\frac{\partial^4\bar{w}}{\partial\bar{t}\,\partial\bar{x}^3}\\
&-\left[\bar{\sigma}+\bar{e}_n^2\left(1+\bar{\beta}_{fe}\delta\left(\bar{x}-1\right)\right)\right]\frac{\partial^4\bar{w}}{\partial\bar{x}^2\partial\bar{t}^2}\\
&+\left[1+\bar{h}-\bar{e}_n^2\left[\bar{v}_f^2\cos\phi-\bar{\xi}\left(\bar{\tau}+\bar{G}\right)\right]\right]\frac{\partial^4\bar{w}}{\partial\bar{x}^4}\\
&-\left[\bar{\mu}\sqrt{\bar{\beta}}+4\bar{e}_n^2\bar{\beta}_f\bar{v}_f\frac{\partial\delta\left(\bar{x}-1\right)}{\partial\bar{x}}\right]\frac{\partial^3\bar{w}}{\partial\bar{x}^2\partial\bar{t}}\\
&+2\bar{v}_f\left[\sqrt{\bar{\beta}}+\bar{\beta}_f\delta\left(\bar{x}-1\right)-\bar{e}_n^2\bar{\beta}_f\frac{\partial^2\delta\left(\bar{x}-1\right)}{\partial\bar{x}^2}\right]\frac{\partial^2\bar{w}}{\partial\bar{t}\,\partial\bar{x}}\\
&+\left[1+\bar{\beta}_{fe}\delta\left(\bar{x}-1\right)\right]\frac{\partial^2\bar{w}}{\partial\bar{t}^2}-\bar{\mu}\bar{v}_f\frac{\partial^3\bar{w}}{\partial\bar{x}^3}\\
&+\left[\bar{v}_f^2\cos\phi-\bar{\xi}\left(\bar{\tau}+\bar{G}\right)-\bar{e}_n^2\bar{K}_w\right]\frac{\partial^2\bar{w}}{\partial\bar{x}^2}+\bar{K}_w\bar{w}=0
\end{aligned}\tag{18}$$

2.7. Solution Method

In this section the Galerkin method is used to solve the equation of motion since it gives better perception to study the vibration and instability of fluid conveying pipes than other numerical methods. To discretize the obtained partial differential equation of motion let:

$$\bar{w}(\bar{x},\bar{t})=\sum_{i=1}^{N}\varphi_i\left(\bar{x}\right)T_i\left(\bar{t}\right),\tag{19}$$

in which $T_i\left(\bar{t}\right)$ is the dynamic response and $\varphi_i\left(\bar{x}\right)$ is the orthogonal function which can be defined for C-F boundary condition as:

$$\begin{aligned}\varphi_i(\bar{x})=&\sin\left(\lambda_i\bar{x}\right)-\sinh\left(\lambda_i\bar{x}\right)+\\&\frac{\sinh\lambda_i-\sin\lambda_i}{\cosh\lambda_i+\cos\lambda_i}\left[\cosh\left(\lambda_i\bar{x}\right)-\cos\left(\lambda_i\bar{x}\right)\right],\end{aligned}\tag{20}$$

in which λ_i is the dimensionless fundamental frequency.

Consequently, substituting Eqs. (19-20) into Eq. (18) and multiplying the obtained relation by φ_j and integrating over the length of the nanotube yields

$$\left[M\right]\frac{\partial^2T_i}{\partial\bar{t}^2}+\left[C\right]\frac{\partial T_i}{\partial\bar{t}}+\left[K\right]T_i=0,\tag{21}$$

where $\left[K\right]$, $\left[C\right]$ and $\left[M\right]$ are the linear stiffness matrix, damping matrix, mass matrix, respectively.

3. Numerical Results and Discussions

In this paper, the free-vibration equation of the fluid-conveying Y-SWCNT has been derived by using nonlocal elasticity theory. The effects of angle of Y-junction, fluid flow, surface tension, nonlocal parameter, Pasternak foundation, are considered. The material properties are: The inner radius $R_i=3.4nm$, the thickness $h=0.34nm$, Young's modulus $E=1TPa$, viscosity $\mu_0=3\times10^{-4}\,Pa.s$, small scale parameter $e_0a=1nm$, $\beta_f=0.01$ and $\beta_{fe}=0.02$.

Fig. 2 demonstrates the influence of the angle of Y-junction angle on the dimensionless flutter velocity where the first and second modes are merged for both C-C and C-F Y-SWCNTs. As can be seen from Fig. 2 increasing the angle of Y-junction enhances the dimensionless flutter velocity at lower natural frequency (first mode) for both C-C and C-F boundary conditions. Hence, the stability of the system increases by using Y-junction at the downstream end of nanotube. Moreover, combination of first and second modes do not occur at near $\phi>38°$ for C-F Y-SWCNTs, while for C-C Y-SWCNTs first and second modes are merged until about $\phi<84°$. Also it is evident that at a given angle of Y-junction the flutter velocity of the C-F Y-SWCNT is higher than C-C one. In order to show the influences of effective flexural rigidity and residual surface stresses on the natural frequency of the system, variation of dimensionless natural and damping frequencies with respect to dimensionless fluid velocity are presented in Figs. 3a and 3b, respectively for C-F Y-SWCNT. As can be seen from Figs. 3 within the zero-frequency area $\bar{U}_d<\bar{U}<\bar{U}_r$ the damping frequency is increased. It is worth mentioning that surface effects depend on the surface crystal orientation and material type. The obtained results from atomistic calculations show that $E^sh^s=35.3N/m$ and

$\tau = 0.31N/m$ for (112) nickel while for (001) nickel $E^s h^s = -43.8N/m$ and $\tau = 0.71N/m$. Also results show that C-F Y-SWCNTs covered by (112) nickel are more stable than those covered by (001) nickel.

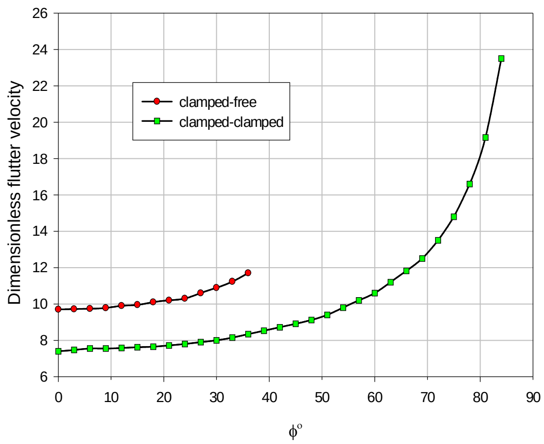

Fig. 2. Effect of the angle of Y-junction on the dimensionless flutter velocity

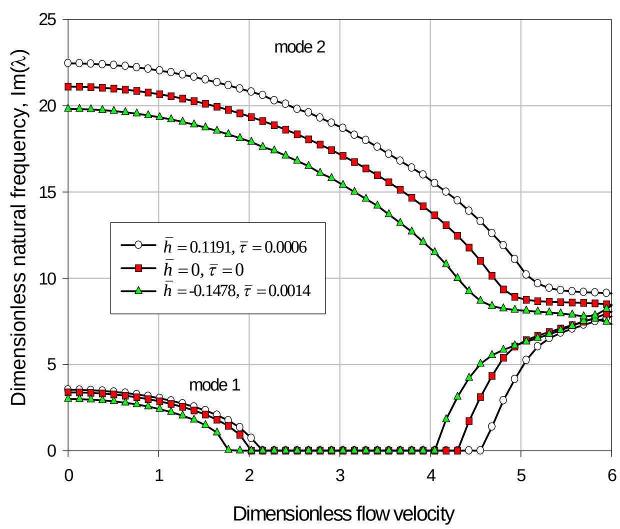

Fig. 3a. Surface effects on the dimensionless natural frequency of C-F Y-SWCNTs

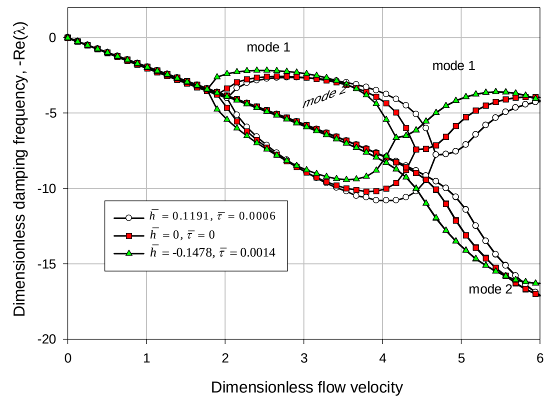

Fig. 3b. Surface effects on the dimensionless damping frequency of C-F Y-SWCNTs

Influences of elastic medium on the dimensionless natural frequency of the Y-SWCNT are demonstrated in Fig. 4 for C-F boundary condition. From Fig. 4 it is observed that before the system becomes unstable at $\bar{U}_d$, considering elastic medium enhances stability and critical fluid velocity. It is also seen that Pasternak foundation more effective than Winkler foundation. This is due to the fact that Winkler foundation describes only the effects of the normal stress of the elastic medium while Pasternak foundation describes the effects of the tangential and normal stresses.

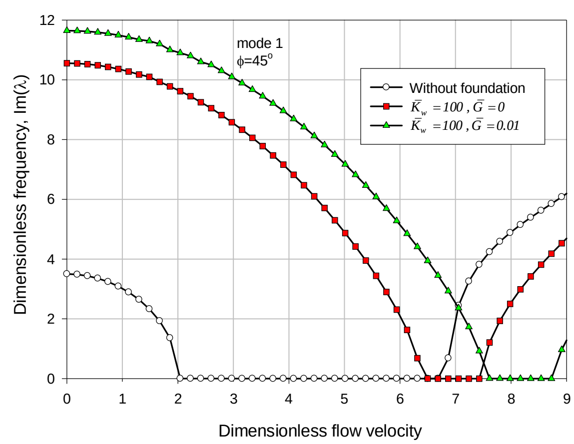

Fig. 4. Effect of the elastic medium on the dimensionless natural frequency

4. Conclusion

In this study, the vibration and instability of a viscoelastic Y-SWCNT conveying fluid embedded on Pasternak is investigated using Euler-Bernoulli beam model considering nonlocal and surface effects. The fluid flow was assumed to be viscose, irrotational, fully developed and isentropic. Regarding fluid flow effects, it has been concluded that the fluid flow is an effective factor on increasing the instability of Y-SWCNT. Furthermore, surface effects depend on the surface crystal orientation and material type.

References

[1] S. Iijima, Nature. 354 (1991) 56-58.
M. Terrones, F. Banhart, N. Grobert, J.C. Charlier, H. Terrones, P.M Ajayan, Physical Review Letters. 89 (2002) 75505-75508.
[2] P. Nagy, R. Ehlich, L.B, Biro, J.Gjyulai, Applied Physics A: Materials Science & Processing .70 (2000) 481-483.
[3] L.A. Chernozatonskii, Physics Letters A. 172 (1992) 173-176.
[4] L.P. Biro, Z.E. Horvath, G.I. Mark, Z. Osvath, A.A. Koos, S. Santucci, J.M. Kenny, Diamond Related Materials. 13 (2004) 241- 249.
[5] A.N. Andriotis, M. Menon, D. Srivastava, L. Chernozatonskii, Physical Review Letters. 87 (2001) 66802-66805.
[6] P. Castrucci, M. Scarselli, M. De Crescenzi, M.A. El Khakani, F. Rosei, N. Braidy, J.H. Yi, Applied Physics Letters (2004) 3857-3859.
[7] P.C. Tsai, Y.R. Jeng, T.H. Fang, Physical Review B. 74 (2006) 45406-45415.
[8] Q, Liu, W, Liu, Z.M. Cui, W.G. Song, L.J. Wan, Carbon. 45 (2007) 268-273.
[9] A.C. Eringen, D.G.B. Edelen, International Journal of Engineering Science. 10 (1972) 233-248.
[10] A.C. Eringen, Nonlocal Continuum Field Theories. Springer-verlag, New York, 2002.
[11] A.C. Eringen, Journal of Applied Physics. 54 (1983) 4703-4710.
[12] A. Ghorbanpour Arani, M, Mohammadimehr, A. Arefmanesh, A. Ghasemi, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 224 (2010) 745-756.
[13] A. Ghorbanpour Arani, M.A. Roudbari, S. Amir, Physica B. 407 (2012) 3646-3653.
[14] T.P. Chang, Computational Materials Science. 54 (2012) 23-27.
[15] Y.D. Kuang, X.Q. He, C.Y. Chen, G.Q. Li, Computational Materials Science . 45 (2009) 875-880.
[16] E. Ghavanloo, F. Daneshmand, M. Rafiei, Physica E. 42 (2010) 2218-2224.
[17] B. Bar On, E. Altus, J of Sound and Vibration. 330 (2011) 652-663.
[18] H.L. Lee, W.J. Chang, Physica E. 43 (2010) 466-469.
[19] M.E. Gurtin, A.I. Murdoch, Archive for Rational Mechanics and Analysis. 57 (1975) 291-323.
L. Wang, Physica E. 43 (2010) 437-439.
[20] Z. Khodami Maraghi, A. Ghorbanpour Arani, R. Kolahchi, S. Amir, M.R. Bagheri, Composites Part B. 45 (2012) 423-432.
[21] C.C. Chern, Y.L. Chen, L.C. Kung, Int J of Computer Mathematics. 87 (2010)1638–64.
[22] G. Chai, C. Fang, X. Gao, Q. Zhao, J of Computational Information Systems. 7 (2011) 507–514.
[23] C.F. Han, C. Zhang, In: Conference on Natural Computation. (2009) 259–64.
[24] D. Bratton, J. Kennedy, In: IEEE Swarm Intelligence Symposium. (2007) 120–127.
[25] M. Pinedo, Scheduling theory, algorithms, and systems. 2nd ed. Prentice Hall 2002.
[26] M. Bazaraa, H. Sherall, C. Shetty, Nonlinear programming, theory and algorithms. New York: Wiley, 1993.
[27] D.S. Norwood, An analysis of Interlaminar Stresses in Unsymmetrically Laminated Plates.

Ph.D. Thesis, Virginia Polytechnic Institute and State University, 1990.

[28] C. Mittelstedt, W. Becker, Archive of Applied Mechanics. 73 (2003) 63–74.