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Forced-Vibration Analysis of a Coupled System of SLGSs by Visco-Pasternak Medium Subjected to a Moving Nano-particle

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Abstract

In this study, forced-vibration analysis of a coupled system of single layered graphene sheets (SLGSs) subjected to the moving nano-particle is carried out based on nonlocal elasticity theory of orthotropic plate. Two SLGSs are coupled with elastic medium which is simulated by Pasternak and Visco-Pasternak models. Using Hamilton's principle, governing differential equations of motion are derived and solved analytically. The effects of small scale, aspect ratio, velocity of nano-particle, time parameter, mechanical properties of graphene sheets, Visco-elastic medium on the maximum dynamic responses of each SLGSs are studied. Results indicate that, if the medium (elastic or visco-elastic medium) of coupled system becomes more rigid, the maximum dynamic displacements of both SLGSs will be closer together.

1. Introduction

In early 2007, the United Nations reported that nanotechnology, which then accounted for approximately 0.1% of the global manufacturing economy, would grow to 14% of the market by 2014. Nanotechnology is a field of applied science concerned with the control of matter at dimensions of roughly 1 to 100 nanometers (nm). At the particle size of 1 to 100 nm, nano-scale materials may have different molecular organizations and properties than the same chemical substances in a larger size. Nano-sized chemicals can have different properties due to increased relative surface area per unit mass, which can increase physical strength and chemical reactivity and also in some cases, the dominance of quantum effects at the nanometer size, which changes basic material properties [1]. Vibration analysis of isotropic and orthotropic plates using the classical theory of elasticity (generalized Hook's law) is stated for various theory of plates in many books [2-4]. The nonlocal elasticity theory was proposed and developed by Eringen [5-8] to consider small scale effect in the continuum model of nano-structures. In recent years, studies about the vibration of nano-structures using the nonlocal theory of elasticity are included many researches due to superior vibration characteristics of them. Pradhan and Phadikar [9] studied the nonlocal vibration of single and double layered nano-plates using the classical and first-order shear deformation (FSDT) theories. The governing differential equations of motion are solved by Navier's approach for simply supported boundary condition. Murmu and Adhikari [10, 11] investigated nonlocal vibration of bonded double nano-plate systems and the governing equations of motion in terms of displacements are solved by the new analytical method.

The graphene sheets are used for manufacturing of many devices such as oscillators, clocks and sensor devices, due to having high resistance and unique properties of them. The application of the single-layered graphene sheet (SLGS) like mass sensors is studied by sakhaee-pour [12, 13]. In recent years, the vibration characteristics of the graphene sheets have attracted attention of many researchers due to their superior vibrational behaviors. Ansari et al. [13] investigated vibrational behavior of SLGS based on the FSDT and the differential equations are solved by the generalized differential quadrature method (GDQ) [14-16] for two different boundary conditions. In the other work, Ansari et al. [17,18] studied vibration of multi-layered graphene sheet (MLGS) using the FSDT of plate. The vibration analysis of orthotropic SLGS using the classical plate theory is carried out by Pradhan and Kumar [19] and the governing equations of motion are solved by the DQM.

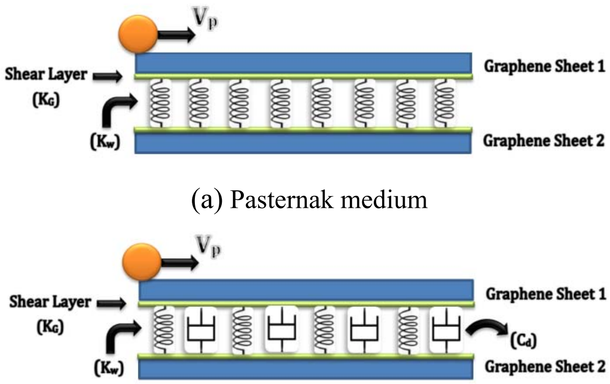
The foundation of sheets can be assumed as linear (Winkler and Pasternak) elastic medium or nonlinear elastic medium. Pradhan and Kumar [20] have carried out vibration analysis of the orthotropic SLGS embedded in a Pasternak elastic medium. The normal forces are considered at the Winkler elastic medium although the shear forces are added also in the Pasternak elastic medium. Behfar and Naghdabadi [21] studied vibration of MLGS embedded in elastic medium. Chien et al. [22] investigated nonlinear vibration of laminated plates on a nonlinear elastic medium. Ghorbanpour et al. [23] studied nonlocal vibration of a coupled system of DLGSs by Visco-Pasternak medium. They used differential quadrature method (DQM) to solve governing equations and investigated effects of different parameters on frequencies of coupled system of DLDS.

Forced vibration of graphene sheets can be assumed under the moving nano-particle. Application of nano-tubes under moving nano-particle is presented to[24]. Kiani is carried out Forced vibration of carbon nano-tube [25] and plates [26-28] subjected to a moving nano-particle and also, Simsek [29] investigated forced vibration of coupled system of carbon nano-tubes under the moving nano-particle. Ghorbanpour et al. [30] studied forced vibration of BNNTs subjected to the moving nano-particle.

Despite of done works that some of them mentioned above, no report has been found in the literature on the forced vibration of the coupled system of SLGSs by the Visco-Pasternak medium subjected to the moving nano-particle. Motivated by this idea, we aim to study forced vibration response of the coupled system of SLGSs by the Visco-Pasternak under the moving nano-particle using the non-local elasticity theory of plate. Governing equations are solved analytically and closed-form solution of dynamic displacement is expressed for each SLGSs on coupled system.

2. Formulation

A schematic diagram of a coupled system of SLGSs by the Pasternak and Visco-Pasternak medium subjected to the moving nano-particle is shown in Fig.1.



(a) Pasternak medium

(b) Visco-Pasternak medium

Fig.1. Coupled system of SLGSs subjected to the moving nano-particle

Geometrical parameters of length *a*, width *b* and thickness *h* are also indicated. Movement velocity of nano-particle is assumed constant velocity (V_p). The transverse loading of moving nano-particle on the upper SLGS can be written as [29]:

$$q_0 = P\delta(x - x_m) \tag{1}$$

where δ is the Delta Dirac function and q_0 is the transverse loading of moving nano-particle and x_m is obtained by:

$$x_m = V_p t \tag{2}$$

where t is the time of arrival nano-particle with constant velocity (V_p) to location x_m.

2.1. General assumptions

The classical laminated plate theory (CLPT) is reformulated by the non-local elasticity theory for considering small scale effect. According to CLPT, the displacement fields can be expressed as [2, 4]

$$
\begin{aligned}
&(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w}{\partial x},\\
&(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w}{\partial y},\\
&(x,y,z,t) = w_0(x,y,t),
\end{aligned} \tag{3}
$$

where u_0, v_0 and w_0 are the displacements along the coordinate lines of a material point on the xy-plane. The small scale effect in the continuum model is considered based on nonlocal elasticity theory that was proposed by Eringen [5,6]. The nonlocal constitutive differential equation of elasticity is expressed as [7, 8]:

$$-\mu\nabla^2)\{\sigma\} = \{t\}, \tag{4}$$

where,

$$= S : \varepsilon, \tag{5}$$

and also, μ is the nonlocal parameter, t is the macroscopic or local stress tensor and σ is the nonlocal stress tensor. The nonlocal stress tensor goes to local stress tensor when the value of the nonlocal parameter goes to zero and also, S is the fourth-order elasticity tensor and (:) denotes the 'double-dot product'. The SLGSs is assumed orthotropic sheet and the matrix S for the orthotropic graphene sheet may be written as [2]

$$S = \begin{pmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{63} \end{pmatrix}, \tag{6}$$

where,

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}c^4 + 2(Q_{12}+2Q_{66})s^2c^2 + Q_{22}s^4,\\
\overline{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})s^2c^2 + Q_{12}(s^4+c^4),\\
\overline{Q}_{22} &= Q_{11}s^4 + 2(Q_{12}+2Q_{66})s^2c^2 + Q_{22}c^4,\\
\overline{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})sc^3\\
&\quad + (Q_{12}-Q_{22}+2Q_{66})cs^3,\\
\overline{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})cs^3\\
&\quad + (Q_{12}-Q_{22}+2Q_{66})sc^3,\\
\overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{66}-2Q_{12})s^2c^2\\
&\quad + Q_{66}(s^4+c^4),
\end{aligned} \tag{7}$$

and also,

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}}, \tag{8}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12} = \frac{E_1}{2(1-\nu_{12})},$$

$$c = \cos\theta, \qquad s = \sin\theta, \tag{9}$$

where the coefficients Q_{ij} (i,j=1,2,6) are known in terms of the engineering constant of the orthotropic graphene sheet, E_1 and E_2 are the Young's module in directions 1 and 2, ν_{12} and ν_{21} denote the Poisson's ratios and G_{12} is the shear modulus. The structure of orthotropic graphene sheet is known armchair (θ=0) and Zigzag (θ=90).

The Von Kármán-type nonlinear strains relations are used in here and these relations are expressed as [2]:

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\\
\varepsilon_{xz} &= \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \quad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),\\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right), \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}.
\end{aligned} \tag{10}$$

The elastic and visco-elastic foundation can be simulated with Pasternak and Visco-Pasternak medium, respectively. Pasternak and Visco-Pasternak loads can be written as [23] :

$$\begin{aligned}
&{}_{Winkler} = K_w W\\
&{}_{Pasternak} = K_w W - K_G \nabla^2 W\\
&{}_{Visco-Winkler} = K_w W + C_d \dot{W}\\
&{}_{Visco-Pasternak} = K_w W - K_G \nabla^2 W + C_d \dot{W}
\end{aligned} \tag{11}$$

where K_w, K_G and C_d are the Winkler, Pasternak and damper modulus parameters, respectively.

2.2. Solving the differential equations of motion

The governing differential equations of motion are derived using the Hamilton's principle which is given as [2]:

$$(\delta U + \delta V - \delta K)dt = 0, \tag{12}$$

where δU is the virtual strain energy, δV is the virtual work done by external applied forces and δK is the virtual kinetic energy. The equations of motion in terms of the displacements for the single layered graphene sheet are derived using Eq. (12) and expressed as [23] :

$$\begin{aligned}
&D_{11}\frac{\partial^4 w}{\partial x^4} - 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}\\
&D_{22}\frac{\partial^4 w}{\partial y^4} + q = I_0\frac{\partial^2 w}{\partial t^2}\\
&I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right)\\
&\mu\nabla^2\left[I_0\frac{\partial^2 w}{\partial t^2} - I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right) - q\right].
\end{aligned} \tag{13}$$

where I_0, I_1, I_2 are mass moments of inertia, ρ_0 denotes the density of the material and D_{ij} can be expressed as:

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 (1, h, h^2) dz, \tag{14}$$

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij} (z^2) dz \quad (i, j = 1, 2, 6). \tag{15}$$

2.2.1. Coupled system of SLGS with Pasternak medium

The motion equations for both SLGSs are derived by substituting Eq.(1) and Eq.(11) into Eq.(13):

SLGS-1 (The upper Sheet):

$$\begin{aligned}
&+\mu K_G \left(\frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial^4 W_1}{\partial x^2 \partial y^2} + \frac{\partial^4 W_1}{\partial y^4} \right) \\
&-\mu K_G \left(\frac{\partial^4 W_2}{\partial x^4} + 2\frac{\partial^4 W_2}{\partial x^2 \partial y^2} + \frac{\partial^4 W_2}{\partial y^4} \right) \\
&+I_2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2} \right) \\
&+\mu \frac{\partial^2}{\partial t^2} \begin{pmatrix} I_0 \left(\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2} \right) \\ -I_2 \left(\frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial^4 W_1}{\partial x^2 \partial y^2} + \frac{\partial^4 W_1}{\partial y^4} \right) \end{pmatrix} = \\
&-P\delta(x - x_m) + \mu P \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta(x - x_m)
\end{aligned} \tag{16}$$

SLGS-2 (The bottom sheet):

$$\begin{aligned}
&-D_{11} \frac{\partial^4 W_2}{\partial x^4} - 2(D_{12} + D_{66}) \frac{\partial^4 W_2}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 W_2}{\partial y^4} \\
&-K_w (W_2 - W_1) + K_G \left(\frac{\partial^2 W_2}{\partial x^2} + \frac{\partial^2 W_2}{\partial y^2} \right) \\
&-K_G \left(\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2} \right) + \mu K_w \left(\frac{\partial^2 W_2}{\partial x^2} + \frac{\partial^2 W_2}{\partial y^2} \right) \\
&-\mu K_w \left(\frac{\partial^2 W_1}{\partial x^2} + \frac{\partial^2 W_1}{\partial y^2} \right) \\
&+\mu K_G \left(\frac{\partial^4 W_2}{\partial x^4} + 2\frac{\partial^4 W_2}{\partial x^2 \partial y^2} + \frac{\partial^4 W_2}{\partial y^4} \right) \\
&-\mu K_G \left(\frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial^4 W_1}{\partial x^2 \partial y^2} + \frac{\partial^4 W_1}{\partial y^4} \right) - I_0 \frac{\partial^2 W_2}{\partial t^2} \\
&+I_2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 W_2}{\partial x^2} + \frac{\partial^2 W_2}{\partial y^2} \right) + \\
&\mu \frac{\partial^2}{\partial t^2} \begin{pmatrix} I_0 \left(\frac{\partial^2 W_2}{\partial x^2} + \frac{\partial^2 W_2}{\partial y^2} \right) - \\ I_2 \left(\frac{\partial^4 W_2}{\partial x^4} + 2\frac{\partial^4 W_2}{\partial x^2 \partial y^2} + \frac{\partial^4 W_2}{\partial y^4} \right) \end{pmatrix} = 0
\end{aligned} \tag{17}$$

For separating the coupled differential equations of motion, W_n is assumed as follow:

$$W_n(x, y, t) = W_1(x, y, t) + W_2(x, y,$$

By summing the coupled differential equations of motion for SLGS-1 and SLGS-2 (Eq.(16) and Eq.(17)), the motion equation in term of W_n can be written as:

$$
\begin{aligned}
&-D_{11}\frac{\partial^4 W_n}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_n}{\partial x^2\partial y^2}+\\
&\mu\frac{\partial^2}{\partial t^2}\left(\begin{array}{l} I_0\left(\dfrac{\partial^2 W_n}{\partial x^2}+\dfrac{\partial^2 W_n}{\partial y^2}\right)\\ -I_2\left(\dfrac{\partial^4 W_n}{\partial x^4}+2\dfrac{\partial^4 W_n}{\partial x^2\partial y^2}+\dfrac{\partial^4 W_n}{\partial y^4}\right)\end{array}\right)\\
&-D_{22}\frac{\partial^4 W_n}{\partial y^4}-I_0\frac{\partial^2 W_n}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 W_n}{\partial x^2}+\frac{\partial^2 W_n}{\partial y^2}\right)\\
&=-P\delta\left(x-x_m\right)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right)
\end{aligned}
\tag{19}
$$

The governing differential equation of motion for SLGS subjected to the moving nano-particle without any foundation derived according to Eq.(19). Based on separation variables the dynamic displacement of SLGS (W_n) for simply supported boundary condition at all edges is considered as:

$$
W_n(x,y,t)=\sum_{i=1}^{n}\sum_{j=1}^{m}q_n^{(i,j)}(t)sin(\frac{i\pi x}{a})sin(\frac{j\pi y}{b})
\tag{20}
$$

By substituting Eq.(20) into Eq.(19), the dynamic displacement of SLGS (W_n) in terms of i and j is derived:

$$
\begin{aligned}
&-D_{11}\frac{\partial^4 W_1}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_1}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 W_1}{\partial y^4}\\
&-K_w(W_1-W_2)+K_G\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)\\
&-K_G\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)+\mu K_w\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)\\
&-\mu K_w\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\
&-I_0\frac{\partial^2 W_1}{\partial t^2}\\
&\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\begin{array}{l}-D_{11}\left(\dfrac{i\pi}{a}\right)^4-D_{22}\left(\dfrac{j\pi}{b}\right)^4\\ -2(D_{12}+D_{66})\left(\dfrac{i\pi}{a}\right)^2\left(\dfrac{j\pi}{b}\right)^2\end{array}\right]\times\\
&q_n\left(t\right)\sin\left(\frac{i\pi y}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m}\mu\left[\begin{array}{l}-I_0\left(\left(\dfrac{i\pi}{a}\right)^2+\left(\dfrac{j\pi}{b}\right)^2\right)\\ -I_2\left(\begin{array}{l}\left(\dfrac{i\pi}{a}\right)^4+\left(\dfrac{j\pi}{b}\right)^4\\ +2\left(\dfrac{i\pi}{a}\right)^2\left(\dfrac{j\pi}{b}\right)^2\end{array}\right)\end{array}\right]\times\\
&\ddot{q}_n\left(t\right)\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)+\\
&\left[-I_0-I_2\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)\right]\times\\
&\ddot{q}_n\left(t\right)\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)
\end{aligned}
\tag{21}
$$

The following terms are defined for reduction or simplification long mathematical sentence. The stiffness matrix of SLGS can be written as:

$$
\begin{aligned}
{}^{(i,j)}&=-D_{11}\left(\frac{i\pi}{a}\right)^4-D_{22}\left(\frac{j\pi}{b}\right)^4\\
&-2(D_{12}+D_{66})\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2
\end{aligned}
\qquad 2)
$$

and also, the mass matrix of SLGS vibration and forced loading of nano-particle can be presented as:

$$m^{(i,j)}=\mu\begin{bmatrix}-I_0\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)\\+I_2\left(\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right)\end{bmatrix}-I_0-I_2\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) \quad (23)$$

$$f_n(\mathrm{x},\mathrm{t})=-P\delta\left(x-x_m\right)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta\left(x-x_m\right) \quad (24)$$

By substituting the stiffness, mass matrix and forced loading of nano-particle into Eq.(21), the differential equation of motion is derived in terms of $q_n(t)$ and its derivatives of that:

$$K^{(i,j)}q_n^{(i,j)}\left(t\right)+m^{(i,j)}\ddot{q}_n^{(i,j)}=f_n(\mathrm{x},\mathrm{t}) \quad (25)$$

Delta Dirac function is removed from Eq.(25), by multiplying both sides of Eq.(25) by $sin(i\pi x/a)sin(j\pi y/b)$ and then double integration of both sides through length and width:

$$\int_0^a\int_0^b\left[K^{(i,j)}q_n^{(i,j)}\left(t\right)+m^{(i,j)}\dot{q}_n^{(i,j)}\right]\times\left(\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)^2dxdy=\int_0^a\int_0^b f_n(\mathrm{x},\mathrm{t})dxdy \quad (26)$$

After performing mathematical operation on the Eq.(25), this equation can be converted to:

$$K_n^{(i,j)}q_n^{(i,j)}\left(t\right)+m_n^{(i,j)}\ddot{q}_n^{(i,j)}=f_n^{(i,j)}(t) \quad (27)$$

where,

$$f_n^{(i,j)}(t)=-\frac{2Pb}{j\pi}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\sin\left(\frac{i\pi V_p t}{a}\right) \quad (28)$$

$$K_n^{(i,j)}=0.25K^{(i,j)}ab \quad (29)$$

$$m_n^{(i,j)}=0.25m^{(i,j)}ab \quad (30)$$

Therefore,

$$\ddot{q}_n^{(i,j)}+\left(\omega_n^{(i,j)}\right)^2q_n^{(i,j)}\left(t\right)=\tilde{Q}_n^{(i,j)}\left(t\right) \quad (31)$$

where,

$$\omega_n^{(i,j)}=\sqrt{\frac{K_n^{(i,j)}}{m_n^{(i,j)}}} \quad (32)$$

$$\tilde{Q}_n^{(i,j)}\left(t\right)=\frac{f_n^{(i,j)}(t)}{m_n^{(i,j)}} \quad (33)$$

$q_n^{(i,j)}$ will obtain by taking Laplace of two sides of Eq.(31),

$$q_n^{(i,j)}\left(t\right)=\frac{1}{\omega_n^{(i,j)}}\int_0^t\tilde{Q}_n^{(i,j)}\left(\tau\right)sin\omega_n^{(i,j)}\left(t-\tau\right)d\tau \quad (34)$$

By substituting Eq.(33) into Eq.(34), $q_n^{(i,j)}$ is obtained:

$$q_n^{(i,j)}\left(t\right)=\frac{1}{\omega_n^{(i,j)}}\int_0^t-\frac{2Pb}{j\pi m_n^{(i,j)}}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\times\sin\left(\frac{i\pi V_p\tau}{a}\right)sin\omega_n^{(i,j)}\left(t-\tau\right)d\tau \quad (35)$$

The velocity parameter can be defined as:

$$r=\frac{\frac{\mathrm{i}\pi \mathrm{v_p}}{\mathrm{a}}}{\omega_n^{(i,j)}} \quad (36)$$

where $i\pi v_p/a$ is frequency of moving nano-particle. Here, two cases for velocity parameter (r) are considered:

1. r=1
2. $r\neq1$

Therefore, for case of $r\neq1$:

$$q_n^{(i,j)}\left(t\right)=\frac{2Pb}{j\pi m_n^{(i,j)}\omega_n^{(i,j)}}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)A_1^{(i,j)}(t) \quad (37)$$

where,

$$A_1^{(i,j)}(t)=\frac{\left(\frac{i\pi V_p t}{a}\right)\sin\left(\omega_n^{(i,j)}t\right)-\omega_n^{(i,j)}\sin\left(\frac{i\pi V_p t}{a}\right)}{\left(\frac{i\pi V_p t}{a}\right)^2-\left(\omega_n^{(i,j)}t\right)^2} \quad (38)$$

and also, the dynamic displacement of SLGS for case of $r\neq1$ can be written as:

$$W_n(x,y,t)=-\frac{2Pb}{j\pi m_n^{(i,j)}}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\times A_1^{(i,j)}(t)\,sin\left(\frac{i\pi x}{a}\right)sin\left(\frac{j\pi y}{b}\right) \quad (39)$$

For case of r=1:

$$q_n^{(i,j)}(t)=\frac{Pb}{j\pi m_n^{(i,j)}\left(\omega_n^{(i,j)}\right)^2}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)A_2^{(i,j)} \quad (40)$$

where,

$$A_2^{(i,j)}=\omega_n^{(i,j)}t\cos\left(\omega_n^{(i,j)}t\right)-\sin\left(\omega_n^{(i,j)}t\right) \quad (41)$$

Therefore, the dynamic displacement of SLGS for case of r=1 can be written as:

$$W_n(x,y,t)=\frac{Pb}{j\pi m_n^{(i,j)}\left(\omega_n^{(i,j)}\right)^2}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\times A_2^{(i,j)}sin\left(\frac{i\pi x}{a}\right)sin\left(\frac{j\pi y}{b}\right) \quad (42)$$

The differential equation of motion for SLGS-2 (Eq.(17)) in term of W_n can be obtained by substituting Eq.(18) into Eq.(17):

$$\begin{aligned}
&-D_{11}\frac{\partial^4W_2}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4W_2}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4W_2}{\partial y^4}\\
&-K_w\left(W_2-W_n(x,y,t)+W_2(x,y,t)\right)+\\
&K_G\left(\frac{\partial^2W_2}{\partial x^2}+\frac{\partial^2W_2}{\partial y^2}\right)+\mu K_w\left(\frac{\partial^2W_2}{\partial x^2}+\frac{\partial^2W_2}{\partial y^2}\right)\\
&-K_G\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(W_n(x,y,t)-W_2(x,y,t)\right)\\
&+\mu K_G\left(\frac{\partial^4W_2}{\partial x^4}+2\frac{\partial^4W_2}{\partial x^2\partial y^2}+\frac{\partial^4W_2}{\partial y^4}\right)\\
&-\mu K_w\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(W_n(x,y,t)-W_2(x,y,t)\right)\\
&-\mu K_G\left(\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}\right)\begin{pmatrix}W_n(x,y,t)\\-W_2(x,y,t)\end{pmatrix}\\
&-I_0\frac{\partial^2W_2}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2W_2}{\partial x^2}+\frac{\partial^2W_2}{\partial y^2}\right)\\
&+\mu\frac{\partial^2}{\partial t^2}\begin{pmatrix}I_0\left(\frac{\partial^2W_2}{\partial x^2}+\frac{\partial^2W_2}{\partial y^2}\right)\\-I_2\left(\frac{\partial^4W_2}{\partial x^4}+2\frac{\partial^4W_2}{\partial x^2\partial y^2}+\frac{\partial^4W_2}{\partial y^4}\right)\end{pmatrix}=0
\end{aligned} \quad (43)$$

The dynamic displacement of SLGS-2 (W_2) for all edges simply supported boundary condition is assumed as:

$$W_2(x,y,t)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)q_2^{(i,j)}(t) \quad (44)$$

By substituting Eq.(44) into Eq.(43), the governing differential equation of motion for SLGS-2 can be written as below:

$$\left[K_2^{(i,j)}q_2^{(i,j)}(t)+m_2^{(i,j)}\dot{q}_2^{(i,j)}(t)\right]\sin\left(\frac{i\pi y}{a}\right)\sin\left(\frac{j\pi y}{b}\right)=f_2^{(i,j)}(t)\,q_n^{(i,j)}(t)\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right) \quad (45)$$

where,

$$K_2^{(i,j)} = -D_{11}\left(\frac{i\pi}{a}\right)^4 - 2(D_{12}+D_{66})\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right) -D_{22}\left(\frac{j\pi}{b}\right)^4 - 2K_w - 2K_G\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) - 2\mu K_w\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) -2\mu K_G\left[\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right] \tag{46}$$

$$m_2^{(i,j)} = \mu\begin{bmatrix} -I_0\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) \\ +I_2\left(\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right)\end{bmatrix} -I_0 - I_2\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) \tag{47}$$

and also,

$$f_2^{(i,j)}(t) = -K_w - K_G\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) -\mu K_w\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) -\mu K_G\left[\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right] \tag{48}$$

Therefore,

$$\ddot{q}_2^{(i,j)}(t) + \left(\omega_2^{(i,j)}\right)^2 q_2^{(i,j)}(t) = \tilde{Q}_2^{(i,j)}(t) \tag{49}$$

The natural frequency of system (coupled system of SLGSs by the Pasternak medium subjected to moving nano-particle) can be obtained as:

$$\omega_2^{(i,j)} = \sqrt{\frac{K_2^{(i,j)}}{m_2^{(i,j)}}} \tag{50}$$

and also,

$$\tilde{Q}_2^{(i,j)}(t) = \frac{f_2^{(i,j)}(t)\, q_n^{(i,j)}(t)}{m_2^{(i,j)}} \tag{51}$$

Therefore, by taking Laplace of two sides of Eq.(49), the below equations are obtained:

$$L\left[\ddot{q}_n^{(i,j)} + \left(\omega_n^{(i,j)}\right)^2 q_n^{(i,j)}(t)\right] = L\left(\tilde{Q}_n^{(i,j)}(t)\right) \tag{52}$$

$$\Rightarrow \left[s^2 Q_n^{(i,j)}(s) + \left(\omega_n^{(i,j)}\right)^2 Q_n^{(i,j)}(s)\right] = L\left(\tilde{Q}_n^{(i,j)}(t)\right) \tag{53}$$

$$\Rightarrow \left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right) Q_n^{(i,j)}(s) = L\left(\tilde{Q}_n^{(i,j)}(t)\right) \tag{54}$$

$$\Rightarrow Q_n^{(i,j)}(s) = \frac{\left[L\left(\tilde{Q}_n^{(i,j)}(t)\right)\right]}{\left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right)} \tag{55}$$

Therefore,

$$q_n^{(i,j)}(t) = L^{-1}(Q_n^{(i,j)}(s)) = L^{-1}\left\{\frac{\left[L\left(\tilde{Q}_n^{(i,j)}(t)\right)\right]}{\left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right)}\right\} = L^{-1}\left(\frac{1}{\left(s^2 + \left(\omega_n^{(i,j)}\right)^2\right)}\right) * \tilde{Q}_n^{(i,j)}(t) \tag{56}$$

$$\Rightarrow q_2^{(i,j)}\left(t\right) = \frac{1}{\omega_2^{(i,j)}}\int_0^t \tilde{Q}_2^{(i,j)}(\tau) sin\omega_2^{(i,j)}\left(t-\tau\right)d\tau \tag{57}$$

For case of r≠1:

$$\tilde{Q}_2^{(i,j)}\left(\tau\right)=\frac{B_1^{(i,j)}}{m_2^{(i,j)}}\times\frac{-2Pb\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{\left(j\pi m_n^{(i,j)}\omega_n^{(i,j)}\right)\left(\left(\frac{i\pi V_p t}{a}\right)^2-\left(\omega_n^{(i,j)}\right)^2\right)}\times\left(\left(\frac{i\pi V_p\tau}{a}\right)\sin\left(\omega_n^{(i,j)}\tau\right)-\omega_n^{(i,j)}\sin\left(\frac{i\pi V_p\tau}{a}\right)\right) \tag{58}$$

where,

$$B_1^{(i,j)}=-K_w-K_G\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)-\mu K_w\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)-\mu K_G\left[\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right] \tag{59}$$

The dynamic displacement of SLGS-2 for r≠1 can be expressed as:

$$W_2(x,y,t)=\frac{\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)Sin\left(\frac{i\pi x}{a}\right)Sin\left(\frac{j\pi y}{b}\right)}{\frac{j\pi m_n^{(i,j)}m_2^{(i,j)}}{-2PbB_1^{(i,j)}}\left(\left(\frac{i\pi V_p}{a}\right)^2-\left(\omega_n^{(i,j)}\right)^2\right)}\times\left(\frac{\frac{\left(\frac{\sin\left(\omega_2^{(i,j)}t\right)}{\omega_2^{(i,j)}}-\frac{\sin\left(\omega_n^{(i,j)}t\right)}{\omega_n^{(i,j)}}\right)}{\left(\omega_n^{(i,j)}-\omega_2^{(i,j)}\right)\left(\omega_n^{(i,j)}+\omega_2^{(i,j)}\right)}}{\left(\frac{i\pi V_p t}{a}\right)}-\frac{\left(\frac{i\pi V_p t}{a}\right)\frac{\sin\left(\omega_2^{(i,j)}t\right)}{\omega_2^{(i,j)}}-\frac{\sin\left(\frac{i\pi V_p t}{a}\right)}{\omega_n^{(i,j)}}}{\left(\frac{i\pi V_p}{a}\right)^2-\left(\omega_n^{(i,j)}\right)^2}\right) \tag{60}$$

Therefore, for case of r=1:

$$\tilde{Q}_2^{(i,j)}\left(\tau\right)=\frac{B_1^{(i,j)}}{m_2^{(i,j)}}\times\frac{Pb}{j\pi m_n^{(i,j)}\left(t\right)\left(\omega_n^{(i,j)}\right)^2}\times\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)\left(\omega_n^{(i,j)}tcos\left(\omega_n^{(i,j)}t\right)-\sin\left(\omega_n^{(i,j)}t\right)\right) \tag{61}$$

The dynamic displacement of SLGS-2 for case of r=1 can be expressed as:

$$W_2(x,y,t)=\left(Pb\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)B_1^{(i,j)}\Big/\left(H^{(i,j)}(t)+G^{(i,j)}(t)-R^{(i,j)}(t)\right)\right)\left(\frac{j\pi m_n^{(i,j)}m_2^{(i,j)}\left(\omega_n^{(i,j)}\right)^2\omega_2^{(i,j)}}{\left(\left(\frac{i\pi V_p}{a}\right)^2-\left(\omega_n^{(i,j)}\right)^2\right)}\right)\times Sin\left(\frac{i\pi x}{a}\right)Sin\left(\frac{j\pi y}{b}\right) \tag{62}$$

where,

$$H^{(i,j)}(t)=-\omega_n^{(i,j)}\times\begin{pmatrix}\left(\omega_2^{(i,j)}\right)^2\sin\left(\omega_2^{(i,j)}t\right)\\+\left(\omega_n^{(i,j)}\right)^2\sin\left(\omega_2^{(i,j)}t\right)\\-2\omega_n^{(i,j)}\omega_2^{(i,j)}\sin\left(\omega_n^{(i,j)}t\right)\end{pmatrix}\Big/\left(\left(\omega_n^{(i,j)}+\omega_2^{(i,j)}\right)^2\left(\omega_n^{(i,j)}-\omega_2^{(i,j)}\right)^2\right) \tag{63}$$

$$G^{(i,j)}(t)=\begin{pmatrix}\left(\omega_n^{(i,j)}\right)^2 t\omega_2^{(i,j)}\cos\left(\omega_n^{(i,j)}t\right)\\-(\omega_n^{(i,j)})^3\cos\left(\omega_n^{(i,j)}t\right)\end{pmatrix}\Big/\left(\left(\omega_n^{(i,j)}+\omega_2^{(i,j)}\right)^2\left(\omega_n^{(i,j)}-\omega_2^{(i,j)}\right)^2\right) \tag{64}$$

$$R^{(i,j)}(t)=\left(\frac{\omega_n^{(i,j)}\sin\left(\omega_2^{(i,j)}t\right)-\omega_2^{(i,j)}\sin\left(\omega_n^{(i,j)}t\right)}{\left(\omega_n^{(i,j)}\right)^2-\left(\omega_2^{(i,j)}\right)^2}\right) \tag{65}$$

2.2.2.Coupled system of SLGSs by the Visco-Pasternak medium

The governing differential equations of motion for each SLGSs of coupled system by the Visco-Pasternak medium are presented as:

SLGS-1:

$$\begin{aligned}&-D_{11}\frac{\partial^4 W_1}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_1}{\partial x^2\partial y^2}\\&-D_{22}\frac{\partial^4 W_1}{\partial y^4}-K_w(W_1-W_2)+K_G\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)\\&-K_G\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)+\mu K_w\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)\\&-\mu K_w\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)-C_d\left(\dot W_1-\dot W_2\right)\\&+\mu K_G\left(\frac{\partial^4 W_1}{\partial x^4}+2\frac{\partial^4 W_1}{\partial x^2\partial y^2}+\frac{\partial^4 W_1}{\partial y^4}\right)\\&-\mu K_G\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\\&+\mu C_d\left(\frac{\partial^2 \dot W_1}{\partial x^2}+\frac{\partial^2 \dot W_1}{\partial y^2}\right)-\mu C_d\left(\frac{\partial^2 \dot W_2}{\partial x^2}+\frac{\partial^2 \dot W_2}{\partial y^2}\right)\\&+\mu\frac{\partial^2}{\partial t^2}\left(I_0\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)-I_2\left(\frac{\partial^4 W_1}{\partial x^4}+2\frac{\partial^4 W_1}{\partial x^2\partial y^2}+\frac{\partial^4 W_1}{\partial y^4}\right)\right)\\&-I_0\frac{\partial^2 W_1}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)=\\&-P\delta(x-x_m)+\mu P\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta(x-x_m)\end{aligned}$$

SLGS-2:

$$
\begin{aligned}
&-D_{11}\frac{\partial^4 W_2}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_2}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 W_2}{\partial y^4}\\
&-K_w(W_2-W_1)+K_G\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)-\\
&K_G\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)+\mu K_w\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\
&-\mu K_w\left(\frac{\partial^2 W_1}{\partial x^2}+\frac{\partial^2 W_1}{\partial y^2}\right)-C_d\left(\dot{W}_2-\dot{W}_1\right)+\\
&\mu K_G\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\\
&-\mu K_G\left(\frac{\partial^4 W_1}{\partial x^4}+2\frac{\partial^4 W_1}{\partial x^2\partial y^2}+\frac{\partial^4 W_1}{\partial y^4}\right)\\
&\times\mu C_d\left(\frac{\partial^2 \dot{W}_2}{\partial x^2}+\frac{\partial^2 \dot{W}_2}{\partial y^2}\right)\\
&-\mu C_d\left(\frac{\partial^2 \dot{W}_1}{\partial x^2}+\frac{\partial^2 \dot{W}_1}{\partial y^2}\right)\\
&\times\mu\frac{\partial^2}{\partial t^2}\left(\begin{array}{l} I_0\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\ -I_2\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\end{array}\right)\\
&-I_0\frac{\partial^2 W_2}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)=0
\end{aligned}
\tag{67}
$$

By substituting Eq.(18) into Eq.(67), the differential motion equation for SLGS-2 in terms of W_n and W_2 is derived:

$$
\begin{aligned}
&-D_{11}\frac{\partial^4 W_2}{\partial x^4}-2(D_{12}+D_{66})\frac{\partial^4 W_2}{\partial x^2\partial y^2}\\
&-D_{22}\frac{\partial^4 W_2}{\partial y^4}-2K_wW_2+2K_G\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\cdot\\
&2\mu K_w\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)-2C_d\left(\dot{W}_2\right)\\
&-2\mu K_G\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\\
&+2\mu C_d\left(\frac{\partial^2 \dot{W}_2}{\partial x^2}+\frac{\partial^2 \dot{W}_2}{\partial y^2}\right)\\
&+\mu\frac{\partial^2}{\partial t^2}\left(\begin{array}{l} I_0\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\ -I_2\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\end{array}\right)\\
&\mu\frac{\partial^2}{\partial t^2}\left(\begin{array}{l} I_0\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\ -I_2\left(\frac{\partial^4 W_2}{\partial x^4}+2\frac{\partial^4 W_2}{\partial x^2\partial y^2}+\frac{\partial^4 W_2}{\partial y^4}\right)\end{array}\right)\\
&-I_0\frac{\partial^2 W_2}{\partial t^2}+I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 W_2}{\partial x^2}+\frac{\partial^2 W_2}{\partial y^2}\right)\\
&=-K_wW_n+K_G\left(\frac{\partial^2 W_n}{\partial x^2}+\frac{\partial^2 W_n}{\partial y^2}\right)\\
&+\mu K_w\left(\frac{\partial^2 W_n}{\partial x^2}+\frac{\partial^2 W_n}{\partial y^2}\right)\\
&-\mu K_G\left(\frac{\partial^4 W_n}{\partial x^4}+2\frac{\partial^4 W_n}{\partial x^2\partial y^2}+\frac{\partial^4 W_n}{\partial y^4}\right)\\
&+\mu C_d\left(\frac{\partial^2 \dot{W}_n}{\partial x^2}+\frac{\partial^2 \dot{W}_n}{\partial y^2}\right)-C_d\left(\dot{W}_n\right)
\end{aligned}
\tag{68}
$$

Therefore,

$$K_{d2}^{(i,j)} q_{d2}^{(i,j)}(t) + C_{d2}^{(i,j)} \dot{q}_{d2}^{(i,j)}(t) + m_{d2}^{(i,j)} \ddot{q}_{d2}^{(i,j)}(t) = f_{d2}^{(i,j)}(t) \quad (69)$$

where,

$$K_{d2}^{(i,j)} = -D_{11}\left(\frac{i\pi}{a}\right)^4 - 2(D_{12}+D_{66})\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2 - D_{22}\left(\frac{j\pi}{b}\right)^4 - 2K_w - 2K_G\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) - 2\mu K_w\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) - 2\mu K_G\left[\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right] \quad (70)$$

$$m_{d2}^{(i,j)} = \mu\left[-I_0\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) + I_2\left(\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4 + 2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right)\right] - I_0 - I_2\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right) \quad (71)$$

$$C_{d2}^{(i,j)} = \left(-2C_d - 2\mu C_d\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)\right) \quad (72)$$

$$f_{d2}^{(i,j)}(t) = B_1^{(i,j)} q_n^{(i,j)}(t) + C_{d2}^{(i,j)} \dot{q}_n^{(i,j)}(t) \quad (73)$$

Derivative of Eq.(37) is obtained as:

$$\dot{q}_n^{(i,j)}(t) = \frac{-2Pb}{j\pi m_n^{(i,j)}\omega_n^{(i,j)}}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right) \times \left(\omega_n^{(i,j)}\left(\frac{i\pi V_p}{a}\right)\cos\left(\omega_n^{(i,j)}t\right) - \omega_n^{(i,j)}\left(\frac{i\pi V_p}{a}\right)\cos\left(\frac{i\pi V_p t}{a}\right)\right) \Big/ \left(\left(\frac{i\pi V_p t}{a}\right)^2 - \left(\omega_n^{(i,j)}\right)^2\right) \quad (74)$$

Using superposition method, $q_{d2}^{(i,j)}$ is obtained by follow relation:

$$q_{d2}^{(i,j)} = q^1{}_{d2}^{(i,j)}(t) + q^2{}_{d2}^{(i,j)}(t) + q^3{}_{d2}^{(i,j)}(t) + q^4{}_{d2}^{(i,j)}(t) \quad (75)$$

where,

$$q^1{}_{d2}^{(i,j)}(t) = \frac{F^{(i,j)}A^{(i,j)}D^{(i,j)}}{\sqrt{\left(K_{d2}^{(i,j)} - m_{d2}^{(i,j)}\left(\omega_n^{(i,j)}\right)^2\right)^2 + \left(C_{d2}^{(i,j)}\omega_n^{(i,j)}\right)^2}} \sin\left(\omega_n^{(i,j)}t + \tan^{-1}\frac{C_{d2}^{(i,j)}\omega_n^{(i,j)}}{K_{d2}^{(i,j)} - m_{d2}^{(i,j)}\left(\omega_n^{(i,j)}\right)^2}\right) \quad (76)$$

$$q^2{}_{d2}^{(i,j)}(t) = \frac{-F^{(i,j)}D^{(i,j)}}{\sqrt{\left(K_{d2}^{(i,j)} - m_{d2}^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2\right)^2 + \left(C_{d2}^{(i,j)}\frac{i\pi V_p}{a}\right)^2}} \sin\left(\frac{i\pi V_p}{a}t + \tan^{-1}\frac{C_{d2}^{(i,j)}\frac{i\pi V_p}{a}}{K_{d2}^{(i,j)} - m_{d2}^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2}\right) \quad (77)$$

$$q^{3\,(i,j)}_{d2}(t)=\frac{-F^{(i,j)}B_3^{(i,j)}}{\sqrt{\left(K_{d2}^{(i,j)}-m_{d2}^{(i,j)}\left(\omega_n^{(i,j)}\right)^2\right)^2+\left(C_{d2}^{(i,j)}\omega_n^{(i,j)}\right)^2}}$$
$$\cos\left(\omega_n^{(i,j)}t+\tan^{-1}\frac{C_{d2}^{(i,j)}\omega_n^{(i,j)}}{K_{d2}^{(i,j)}-m_{d2}^{(i,j)}\left(\omega_n^{(i,j)}\right)^2}\right)\tag{78}$$

$$q^{4\,(i,j)}_{d2}(t)=\frac{F^{(i,j)}B_3^{(i,j)}}{\sqrt{\left(K_{d2}^{(i,j)}-m_{d2}^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2\right)^2+\left(C_{d2}^{(i,j)}\frac{i\pi V_p}{a}\right)^2}}$$
$$\cos\left(\frac{i\pi V_p}{a}t+\tan^{-1}\frac{C_{d2}^{(i,j)}\frac{i\pi V_p}{a}}{K_{d2}^{(i,j)}-m_{d2}^{(i,j)}\left(\frac{i\pi V_p}{a}\right)^2}\right)\tag{79}$$

where,

$$F^{(i,j)}=\frac{\frac{-2Pb}{j\pi m_n^{(i,j)}}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)}{\left(\frac{i\pi V_p}{a}\right)^2-\left(\omega_n^{(i,j)}\right)^2}\tag{80}$$

$$A^{(i,j)}=\left(\frac{i\pi V_p}{a\omega_n^{(i,j)}}\right)\tag{81}$$

$$D^{(i,j)}=$$
$$-K_w-K_G\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)-\mu K_w\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)$$
$$-\mu K_G\left(\left(\frac{i\pi}{a}\right)^4+\left(\frac{j\pi}{b}\right)^4+2\left(\frac{i\pi}{a}\right)^2\left(\frac{j\pi}{b}\right)^2\right)\tag{82}$$

$$B_3^{(i,j)}=\omega_n^{(i,j)}\left(\frac{i\pi V_p}{a}\right)\left(-\mu C_d\left(\left(\frac{i\pi}{a}\right)^2+\left(\frac{j\pi}{b}\right)^2\right)-C_d\right)\tag{83}$$

Therefore, the dynamic displacement of SLGS-2 is derived as:

$$W_2(x,y,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\begin{array}{l}q^{1\,(i,j)}_{d2}(t)+q^{2\,(i,j)}_{d2}(t)\\+q^{3\,(i,j)}_{d2}(t)+q^{4\,(i,j)}_{d2}(t)\end{array}\right)\times\left(\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)\right)\tag{84}$$

The phase of system is obtained as:

$$\varphi_1=\tan^{-1}\left(\frac{2\xi r_1}{1-r_1^2}\right),\varphi_2=\tan^{-1}\left(\frac{2\xi r_2}{1-r_2^2}\right)\tag{85}$$

where,

$$r_1=\frac{\omega_n^{(i,j)}}{\omega_2^{(i,j)}}\ ,\quad r_2=\frac{\frac{i\pi V_p}{a}}{\omega_2^{(i,j)}},$$
$$\xi=\frac{C_{d2}^{(i,j)}}{2m_{d2}^{(i,j)}\omega_2^{(i,j)}}\tag{86}$$

For cases of r=1, derivative of Eq.(40) is obtained as:

$$\dot{q}_n^{(i,j)}(t)=\frac{Pb}{j\pi m_n^{(i,j)}(t)\left(\omega_n^{(i,j)}\right)^2}\left(1+\mu\left(\frac{i\pi}{a}\right)^2\right)$$
$$\times\left(\begin{array}{l}-\left(\omega_n^{(i,j)}\right)^2 t\,sin\left(\omega_n^{(i,j)}t\right)-\omega_n^{(i,j)}\cos\left(\omega_n^{(i,j)}t\right)\\+\omega_n^{(i,j)}cos\left(\omega_n^{(i,j)}t\right)\end{array}\right)\tag{87}$$

Using the superposition method W2 for case of r=1 is obtained.

3. Result and discussions

The values of non-local parameter, Winkler, Pasternak and damper modulus parameter and mechanical properties of orthotropic and isotropic graphene sheet are taken according to [23]. The maximum static displacement of plate subjected to a concentrated force on the middle of plate is derived according to [3]:

$$W_{st}^{\max} = \frac{4P}{\pi^4 abD}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\left(\frac{i}{a}\right)^2+\left(\frac{j}{b}\right)^2\right)^{-2} \quad (88)$$

were,

$$D = \frac{Eh^3}{12(1-\vartheta^2)} \quad (89)$$

The frequency ratio of system is derived by:

$$FR = \frac{\left(\omega_n^{(i,j)}\right)^{non-local}}{\left(\omega_n^{(i,j)}\right)^{local}} \quad (90)$$

where, the non-local and local natural frequency are derived by substituting $\mu \neq 0$ and $\mu = 0$ into Eq.(32), respectively. As regards validation of our work, the SLGS frequency ratio can be calculated from Eq.(90), and considering $K_w = K_G = C_d = 0$. However, the obtained results for the selected values of non-local parameter are listed in Table 1. As can be seen, the frequency ratio with increasing non-local parameter decreases and the results in this paper are in good agreement with previous researches.

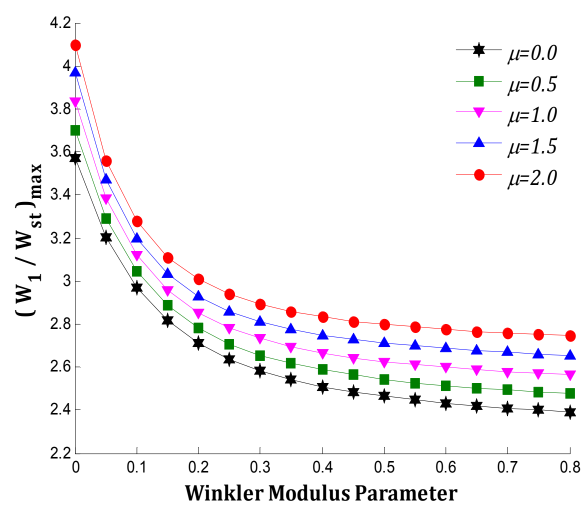
Table 1. Validation results

Nonlocal Parameter (μ) nm^2	Ref. [19, 20] ω^{nl}/ω^{l}	Ref. [9] ω^{nl}/ω^{l}	Ref.e [23] ω^{nl}/ω^{l}	Present paper ω^{nl}/ω^{l}
0	1.0000	1.0000	1.000000	1.0000
1	0.9139	0.9139	0.913865	0.9139
2	0.8467	0.8468	0.846733	0.8468
3	0.7925	0.7926	0.792509	0.7926

Here, the maximum dynamic displacement to static displacement of sheets is shown in diagrams as (W_i / W_{st}) and is called dimensionless displacement.

The maximum dimensionless displacement of SLGS-1 and SLGS-2 with respect to the Winkler modulus parameter for various values of non-local parameter is presented in Fig. 2. It can be seen that, the effect of Winkler modulus parameter on the maximum dynamic displacement for SLGS-1 and SLGS-2 is different with each other, so that by increasing values of Winkler modulus parameter, the maximum dynamic displacement for SLGS-1 decrease whereas, for SLGS-2 increases. For validation this work, it can be expressed that, the effects of Winkler modulus parameter on the maximum dynamic displacement of SLGSs are in good agreement with performed previous research on the carbon nano-tube [29].

(a)



(b)

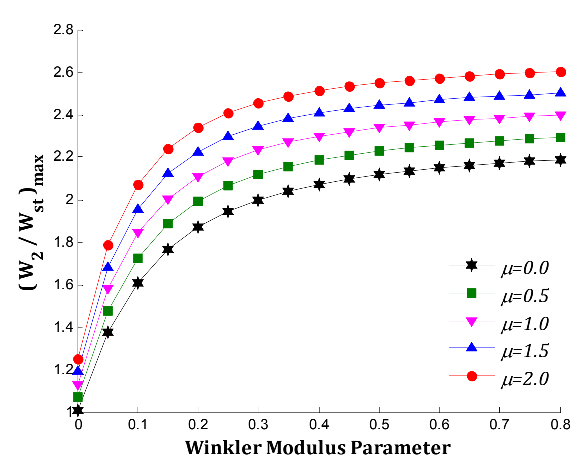
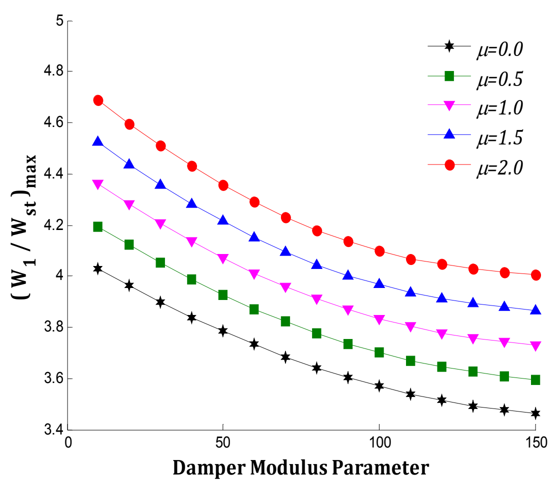


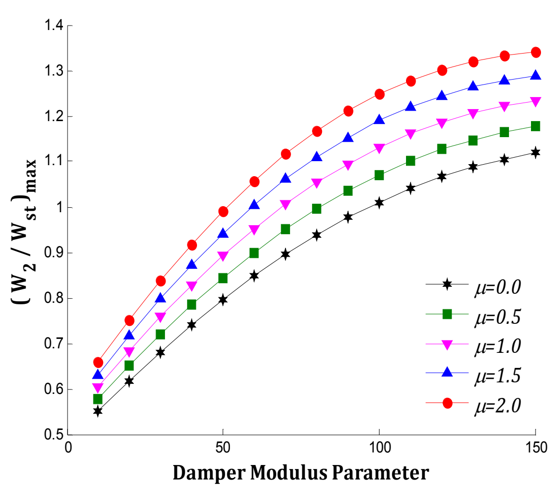
Fig. 2. The maximum dimensionless displacement of coupled SLGSs versus the Winkler modulus parameter (a) SLGS-1 (b) SLGS-2

The maximum dimensionless displacement versus the Pasternak modulus parameter for various values of non-local parameter is illustrated in Fig.3. It is evident that, as the values of Pasternak modulus parameter increase, the maximum dynamic deflection for SLGS-1 reduces. Also, the maximum dynamic displacement of SLGS-2 by increasing Pasternak modulus parameter increases.

(a)



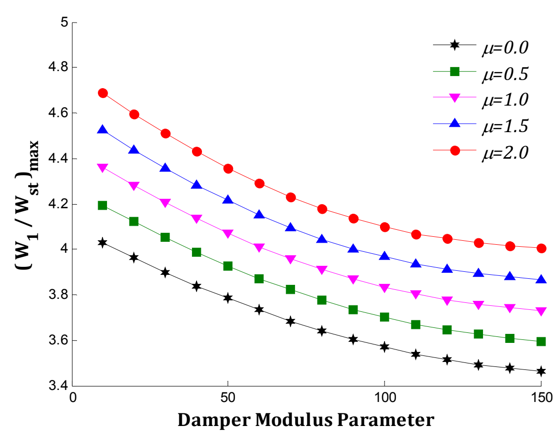
(b)



. The maximum dimensionless displacement of coupled SLGSs versus the shear modulus parameter (a) SLGS-1 (b) SLGS-2

The effect of damper modulus parameter on the maximum dynamic displacement for various values of non-local parameter is shown in Fig. 3. The effect of damper modulus parameter on the maximum dynamic responses of SLGS-1 and SLGS-2 is similar to Winkler and Pasternak modulus parameter effects. The effects of Winkler, Pasternak and damper modulus parameter on the dynamic deflections of SLGSs are similar together, although, the amount of effect for each parameter is different together. Comparing the influences of Winkler, Pasternak and damper modulus parameter on the maximum dynamic displacement, it can be concluded that the effect of Winkler modulus parameter is higher than Pasternak modulus parameter on dynamic displacement of SLGSs and can be expressed as:

(a)



(b)

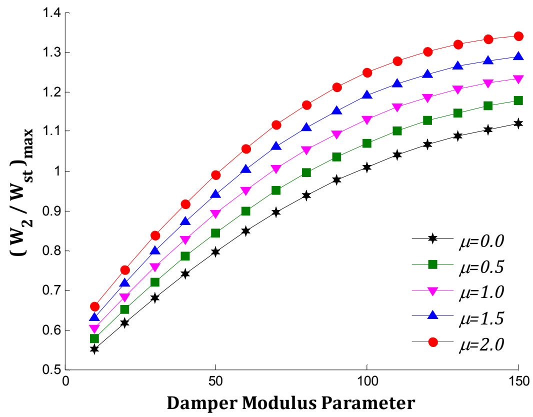


Fig. 3. The dimensionless displacement with respect to the damper modulus parameter for (a) SLGS-1 (b) SLGS-2

The natural frequency of system for isotropic and orthotropic structure of SLGSs is compared in Table 2. It can be seen, the natural frequency of orthotropic structure of SLGSs is higher than that isotropic structure therefore, can be expressed that, the orthotropic structure of SLGSs is more rigid than isotropic structure of that in coupled system of SLGSs. The maximum dynamic displacement of SLGSs with respect to damper modulus parameter for various structures of that is plotted in Fig.5. It can be observed that, the maximum dynamic displacements of orthotropic SLGSs is lower than that isotropic SLGSs for all values of non-local parameter, perhaps it is due to the stiffness of orthotropic structure is more than that isotropic SLGSs.

Error! Reference source not found. Fig. 5. The effect of damper constant on maximum dimensionless displacement of (a) SLGS-1 (b) SLGS-2.

Table 2. Comparing the natural frequency of system for isotropic and orthotropic structure

Non-local Parameter (nm^2)	Isotropic Structure (THz)	Orthotropic Structure (THz)
0.0	0.3777	0.4656
1.0	0.3523	0.4313
2.0	0.3328	0.4049
3.0	0.3174	0.3838

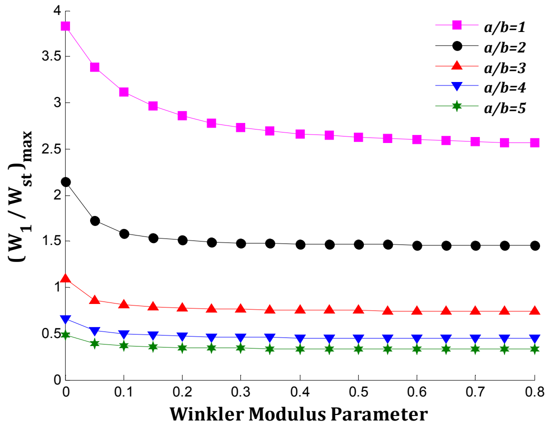
The maximum dynamic displacements of various systems are compared in Table 3. all assumed systems are considered subjected to moving nano-particle: System 1: SLGS without any foundation. System 2: Coupled system of SLGSs by the Visco-Pasternak medium. (The maximum stiffness is considered for coupled medium. In other words, the maximum values of Winkler, Pasternak and damper modulus parameter for medium between SLGSs are investigated.) System 3. Double-layered of graphene sheet with linear vdW force between two sheets.

Table 3. Comparing the maximum dynamic displacement of SLGS, Coupled System of SLGSs, Double-Layered Graphene Sheet

Non-local Parameter	System 1	System 2	System 3
0.0	4.5804	2.2291	2.2896
0.5	4.7720	2.3281	2.3855
1.0	4.9638	2.4267	2.4814
1.5	5.1556	2.5250	2.5773
2.0	5.3474	2.6229	2.6732
2.5	5.5390	2.7204	2.7691
3.0	5.7308	2.8178	2.8650

It can be observed that, the maximum dynamic displacements of system 2 and 3 are equal to each other, approximately. Also, the results indicate that, the maximum dynamic displacements of system 2 and 3 almost are half value of that for system 1. It is due to the rigidity (bending stiffness) of system 2 and 3 is twice of that for system 1.

(a)



(b)

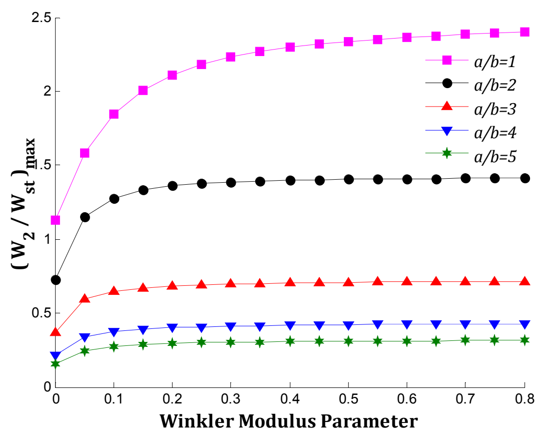


Fig. 6. The effect of aspect ratio (a/b) on the maximum dynamic displacement for (a) SLGS-1 (b) SLGS-2

The effect of aspect ratio of SLGSs (a/b) on the maximum dynamic displacement for various Winkler modulus parameter is plotted as a diagram in Fig. . The results indicate that by increasing the aspect ratio of SLGSs, the effect of Winkler modulus parameter (stiffness of medium) on the maximum dynamic displacement decreases. In other words, by increasing the scale of SLGSs, the effect of moving nano-particle and medium stiffness coefficient on SLGSs reduces.

4. Conclusion

In this paper, forced-vibration analysis of a coupled system of SLGSs by the Visco-Pasternak medium subjected to a moving nano-particle is performed by the non-local elasticity theory of orthotropic plate. The Hamilton's principle is used for deriving the governing differential equations of motion. Moving the nano-particle on upper SLGS is assumed as the linear movement with constant velocity from an edge of SLGS to another edge. The governing differential equations of motion are solved by an analytical method and the closed-form solution of dynamic displacement of SLGSs is presented. The effect of various parameters such as: non-local parameter, Winkler, Pasternak, damper modulus parameters, isotropic and orthotropic structures of sheets, aspect ratio of SLGSs (a/b), velocity and time parameter are discussed and compared with each other. From this work following conclusions are drawn:

1. The maximum dynamic displacement of upper sheet (SLGS-1) by increasing values of Winkler, Pasternak and damper modulus parameter decrease. Whereas, as the visco-elastic medium between two sheets becomes more rigid (strong coupled medium), the maximum dynamic responses of bottom sheet (SLGS-2) increase.
2. The dynamic deflections derived by the classical theory of plate ($\mu = 0$) for both of SLGSs are smaller than those derived by the non-local theory ($\mu \neq 0$). In other words, as the values of non-local parameter increase, the dynamic responses of SLGSs decrease. It is due to small scale effect on the dynamic displacement of SLGSs.
3. The natural frequency of coupled system with orthotropic structure is more than that with isotropic structure. In other words, the rigidity of isotropic structure in this system less than orthotropic structure of that. Therefore, the

dynamic deflections of orthotropic SLGSs are smaller than those isotropic SLGSs.

4. By increasing the stiffness of medium between two sheets, the values of dynamic displacement of SLGS-1 decrease, so that, the values of that for SLGS-2 increase. If the stiffness of medium between two sheets is considered maximum of that, the maximum deflections of SLGSs are equal with each other just like a double-layered of SLGS. The maximum dynamic deflections of SLGS are twice of that for coupled system of SLGSs and double-layered SLGS. It is due to rigidity of coupled system of SLGSs and double-layered SLGS is half of that for SLGS.

5. The presented closed-form solutions of dynamic deflection for SLGSs are very useful to study dynamic behavior of coupled system or double-layered of SLGSs subjected to moving nano-particle.

References

[1] K. Sellers, C. Mackay, L.L. Bergeson, S.R. Clough, M. Hoyt, J. Chen, K. Henry, J. Hamblen, Nanotechnology and the Environment, CRC Press, 2008.

[2] J.N. Reddy, Mechanics of laminated composite plates and shells, CRC press LLC, New York, 2003.

[3] S. Timoshenko, Theory Of plates and shells, Secound Edition ed., McGraw-Hill, 1959.

[4] J.R. VINSON, Plate and Panel Structures of Isotropic, Composite and Piezoelectric Materials, Including Sandwich Construction, Springer, Netherlands., 2005.

[5] A.C. Eringen, Int. J. Eng. Sci., 10 (1972) 1-16.

[6] A.C. Eringen, J. Appl. Phys., 54 (1983) 4703–4710.

[7] A.C. Eringen, Nonlocal Continuum Field Theories, Springer-Verlag, New York, 2002.

[8] A.C. Eringen, D.G.B. Edelen, Int. J. Eng. Sci., 10 233–248.

[9] S.C. Pradhan, J.K. Phadikar, J. Sound Vib., 325 (2009) 206-223.

[10] T. Murmu, S. Adhikari, Composites Part B, 42 (2011) 1901-1911.

[11] T. Murmu, S.C. Pradhan, Physica E, 41 (2009) 1628-1633.

[12] A. Sakhaee-Pour, M.T. Ahmadian, A. Vafai, Solid State Commun., 145 (2008) 168-172.

[13] R. Ansari, S. Sahmani, B. Arash, Phys. Lett. A, 375 (2010) 53-62.

[14] C. shu, Diffrential quadrature and its application in engineering, springer, 1999.

[15] C. Shu, H. Du, Compos. Technol. Part B., 28 (1997) 267.

[16] Z. Zong, Y. Zhang, Advanced Diffrential Quadrature Methods, CRC Press, New York, 2009.

[17] R. Ansari, B. Arash, H. Rouhi, Compos. Struct., 93 (2011) 2419-2429.

[18] R. Ansari, R. Rajabiehfard, B. Arash, Comput. Mater. Sci., 49 (2010) 831-838.

[19] S.C. Pradhan, A. Kumar, Compos. Struct., 93 (2011) 774-779.

[20] S.C. Pradhan, A. Kumar, Comput. Mater. Sci., 50 (2010) 239-245.

[21] K. Behfar, R. Naghdabadi, Compos. Sci. Technol., 65 (2005) 1159-1164.

[22] R.-D. Chien, C.-S. Chen, Compos. Struct., 70 (2005) 90-99.

[23] A. Ghorbanpour Arani, A. Shiravand, M. Rahi, R. Kolahchi, Physica B, 407 (2012) 4123-4131.

[24] K. Kiani, H. Ghaffari, B. Mehri, Curr. Appl Phys.,13(2012) 107-120.

[25] K. Kiani, Physica E, 42 (2010) 2391-2401.

[26] K. Kiani, J. Sound Vib., 330 (2011) 4896-4914.

[27] K. Kiani, Physica E, 44 (2011) 229-248.
[28] K. Kiani, Physica E, 44 (2011) 249-269.
[29] M. Şimşek, Comput. Mater. Sci., 50 (2011) 2112-2123.
[30] A. Ghorbanpour Arani, M.A. Roudbari, S. Amir, Physica B, 407 (2012) 3646-3653.