

**ORIGINAL RESEARCH PAPER**

## **Study the Surface Effect on the Buckling of Nanowires Embedded in Winkler–Pasternak Elastic Medium Based on a Nonlocal Theory**

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### **ABSTRACT**

Nano structures such as nanowires, nanobeams and nanoplates have been investigated widely for their innovative properties. In this paper the buckling of nanowires surrounded in a Winkler - Pasternak elastic medium has been examined based on the nonlocal Euler-Bernoulli model with considering the surface effects. In the following a parametric study that explores the influence of numerous physical and geometrical parameters on the buckling of nanowires is presented. It has been shown that by growing the ratio of surface area to bulk in nano-size structures, the effect of surface energy turn out to be important and should be taken into consideration. Moreover the results point out that surface elasticity and residual surface tension stimulus the buckling behavior of nanowires.

### **INTRODUCTION**

Nanowires and nanotubes have numerous industrially significant applications. For such usages, the precise clarification of the mechanical properties of nanostructures is a key problem. At nano dimension, because of the great ratio of surface area to volume, mechanical and physical properties of nanowire expose new features as surface effects and size effects. Therefore the essential question the researchers pursue to address is what the role of surface effect is in the reaction of nanowires to mechanical loads [1, 2].

The AFM experimental results have shown that the mechanical properties of nanostructures are size-dependent; moreover this concept has been theoretically clarified by allowing for the surfaces effects. The earlier investigations have been revealed that effects of surface have a key role in the mechanical behavior of nanostructures. Besides, nanowires in such cases requisite to undergo an axial load devoid of buckling. In recent years, a new technique based on the conventional Euler buckling model has been

suggested to specify the elastic modulus of nanowires by computing its critical force of buckling [3]. Riaz et al. using a nanoindentation technique studied the effect of surface energy on the instability and buckling of ZnO nanowire [4]. Ansari and Sahmani proposed a non-classical solution to investigate bending and buckling responses of nanobeams including surface stress effects [5]. Hasheminejad et al. studied flexural vibrations of cracked micro- and nanobeams in the presence of surface effects [6]. Rahmani and Noroozi Moghaddam considered the surface effects to study the electromechanical coupling behavior of piezoelectric Nano-beams. In this work the exact solution for free vibration was derived for the simply supported boundary conditions [7]. Challamel and Elishakoff pointed out the role of boundary conditions in the buckling and vibration response of small scale beams in presence of surface elasticity effects [8].

Recently many nanotechnologists use the nonlocal elasticity in modeling the mechanical behavior of nanostructures. Based on the Eringen's nonlocal elasticity theory, the stress at a point in the structure is

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considered to be dependent not only on the strain at this point but also on the strain at all of the points in the structure. Therefore, the nonlocal model covers data about long range loads among atoms; besides the interior length scale is presented into the constitutive relations basically as material factor to capture the small scale influence [9]. Reddy reformulated various beam theories, including the Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories, using the nonlocal differential constitutive relations of Eringen [10]. The equations of motion of the nonlocal theories were derived, and variational statements in terms of the generalized displacements were presented. Rahmani studied the flexural vibration of pre-stressed nanobeams based on nonlocal theory [11]. It was shown that increase of the axial compressive loading leads to decrease of the fundamental frequency of nanobeams. Pirmohammadi et al. investigated the active vibration suppression of a single-walled carbon nanotube under the action of a moving harmonic load using Eringen’s nonlocal elasticity theory [12]. Rahmani and Ghaffari studied the free vibration of Nano-sandwich-structure with nonlocal effect. The model allows for the flexibility of the sandwich core while the faces were treating as beams [13].

From the literature review, it is observed that the nonlocal and surface effects on the mechanical behavior of nanostructures have been investigated separately. On the other hand, the previous studies showed that both these effects have important roles on the behaviors of nanostructures. For example Lee and Chang showed that the frequency ratio of the nanocantilever beam is sensitive to both the surface and nonlocal effects [14]. When the nonlocal effect was taken into account without consideration of surface effects, the frequency ratio of the beam decreased with decrease in width ratio and with increase in mode number. However, the situation was reversed when the surface effects were taken into account without consideration of nonlocal effect. Consequently, to perform an accurate vibration analysis, the formulation should include both these effects. Only a few number of studies investigated the surface and nonlocal effects together [15–18]. Chen et al. formulated a theoretical outline to survey the size effect due to both nonlocal effect and interface effect for a composite material and found that both nonlocal and surface effects dominate the size-dependent effective property of the material on nanoscale [15].

Mahmoud et al. considered the coupled effects of surface properties and nonlocal elasticity on the static deflection of nanobeams [18]. Surface elasticity was applied to describe the behavior of the surface layer. Information about the forces between atoms, and the internal length scale were proposed by the nonlocal Eringen model. Lee and Change used Rayleigh–Ritz method to analyze the influences of surface and nanolocal effects on the critical buckling load of the nonuniform nanowire [17]. Eltaher et al. studied the coupled effects of surface properties and nonlocal elasticity on vibration characteristics of nanobeams by using FEM [16]. Nonlocal differential elasticity of Eringen was exploited to reveal the long-range interactions of a nanoscale beam. To incorporate surface effects, Gurtin–Murdoch model was proposed to satisfy the surface balance equations of the continuum surface elasticity.

In the present paper, based on the Eringen’s nonlocal constitutive relations and by applying the effects of surfaces, equilibrium equation of nanowire surrounded in an elastic medium is achieved. Also the elastic medium has been modeled as Winkler–Pasternak foundation. Winkler elastic foundation comprises of narrowly spaced linear elastic springs which are continuously integrated at the bottom beam surface. The foundation parameter is defined by the springs stiffness and the response force of substance taken to be linearly proportional to the beam deflection [19, 20]. The Winkler model does not make allowance for the continuousness of the elastic environment. A more precise model of the elastic foundation can be obtained by a two parameters elastic foundation recognized as the Winkler–Pasternak model. The Pasternak foundation model simulates the transverse shear stress as a result of the shear deformation of the medium [21], whereas the Winkler approach accounts for the normal pressure from the surrounding medium [22]. In conclusion a closed-form solution of buckling load is achieved for simply supported nanowire and a parametric study is presented to investigate the effect of geometrical properties such as diameter and length of the nanowire.

## THEORETICAL FORMULATION

Consider a nanowire as shown in Fig. 1. The equilibrium equation based on the nonlocal Euler–Bernoulli model [10] is as following:

$$\frac{\partial^2}{\partial x^2} \left( -EI \frac{\partial^2 w}{\partial x^2} \right) + \mu \frac{\partial^2}{\partial x^2} \left( \frac{\partial}{\partial x} \left( p \frac{\partial w}{\partial x} \right) - q \right) + q - \frac{\partial}{\partial x} \left( p \frac{\partial w}{\partial x} \right) = 0 \quad (1)$$

$$q(x) = H \frac{\partial^2 w}{\partial x^2} + f \quad (5)$$

where  $w(x)$  represents the deflection at point  $x$ ,  $p$  is the axial compressive load and  $q$  is the transverse distributed forces of the nanowire.

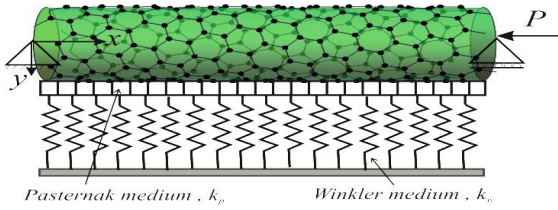


Fig.1. Buckling of a circular nanowire under axial load.

In the case of a circular cross section nanowire,  $EI$  is presented as [10]:

$$EI = E \pi D^4 / 64 \quad (2)$$

where  $E$  and  $D$  are the Young's modulus and diameter of a nanowire. The influence of surface elasticity on the buckling of a nanowire can be applied by changing the traditional flexural rigidity  $EI$  for the bulk material by the effective flexural rigidity  $\bar{EI}$  for a nanowire, which is given by:

$$\bar{EI} = \frac{1}{64} \pi E D^4 + \frac{1}{8} \pi E^s D^3 \quad (3)$$

Where  $E^s$  is the surface modulus. In the following, the vibration and buckling of nanowire based on this surface model will be considered. The residual surface tension will produce a distributed transverse loading  $q(x)$  lengthwise the longitudinal direction. In the present study, axial load caused by elastic medium is supposed in the following form based on the Winkler and Pasternak foundations:

$$f = k_w w + k_p \frac{\partial^2 w}{\partial x^2} \quad (4)$$

where  $k_w$  (nN/nm) and  $k_p$  (nN/nm) are the Winkler and Pasternak stiffness parameters of the elastic medium. The distributed transverse loading produced by the residual surface tension and axial force owing to elastic medium is:

where the factor  $H$  is a constant calculated by the remaining surface tension and the shape of the cross section.  $H$  is specified, individually, by [10]:

$$H = 2\tau^0 D \quad (6)$$

where  $\tau^0$  is the residual surface tension. Consequently putting Equations (3-6) in Equation (8) leads to following equilibrium equation for a nanowire embedded in an elastic medium:

$$\left( -\bar{EI} + \mu p - H\mu - k_p \mu \right) \frac{\partial^4 w}{\partial x^4} + \left( H + k_w \mu - p + k_p \right) \frac{\partial^2 w}{\partial x^2} - k_w w = 0 \quad (7)$$

When  $k_p=0$  and  $k_w=0$ , Equation (7) is returned to the nanowire equilibrium equation without an elastic medium. To investigate the buckling of a nanowire embedded in an elastic medium Equation (7) should be solved for particular boundary conditions.

### ANALYTICAL SOLUTIONS OF BUCKLING OF SIMPLY SUPPORTED BEAMS

In this section exact solutions of buckling of simply supported nanowire will be considered. The boundary conditions are expressed as:

$$w = 0 \text{ and } M = 0 \text{ at } x = 0, L \quad (8)$$

Here we evaluate the critical buckling force for nanowire. The following expansions of the displacements  $w(x)$  fulfill the boundary conditions in Equation (8) and the supposed buckling mode is specified as:

$$w(x, t) = \sum_{n=1}^{\infty} W_n \sin \left( \frac{n\pi}{L} x \right) e^{i\omega_n t} \quad (9)$$

When  $t = 0$ , Thus we have

$$w(x, 0) = \sum_{n=1}^{\infty} W_n \sin \left( \frac{n\pi}{L} x \right) \quad (10)$$

Applying the supposed displacement mode in the equilibrium equation (Equation (7)) result in:

$$\begin{aligned} & (-\overline{EI} + \mu p - H\mu - k_w \mu) n^4 \frac{\pi^4}{L^4} \sin\left(\frac{n\pi}{L}\right) x - \\ & (H + k_w \mu - p + k_p) n^2 \frac{\pi^2}{L^2} \sin\left(\frac{n\pi}{L}\right) x \\ & - k_w \sin\left(\frac{n\pi}{L}\right) x = 0 \end{aligned} \quad (11)$$

As a final point the buckling force is achieved as:

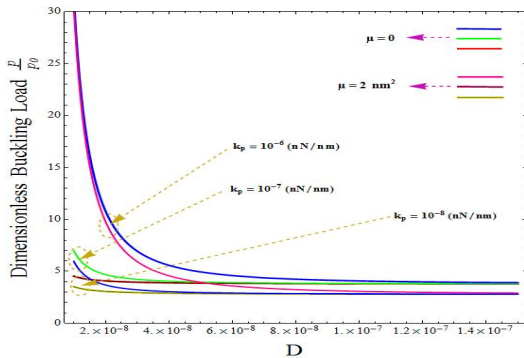
$$p_{cr} = \frac{(\overline{EI} + H\mu + k_p \mu) n^4 \frac{\pi^4}{L^4} + (H + k_w \mu + k_p) n^2 \frac{\pi^2}{L^2} + k_w}{\left( \mu n^4 \frac{\pi^4}{L^4} + n^2 \frac{\pi^2}{L^2} \right)} \quad (12)$$

On the other hand, by solving Equation (1) and using the boundary conditions at the two ends of the beam, correspondingly, the critical axial load of a wire without considering the elastic medium and surface effects is obtained as

$$P_{cro} = EI \frac{\pi^2}{L^2} \quad (13)$$

## NUMERICAL RESULTS AND DISCUSSION

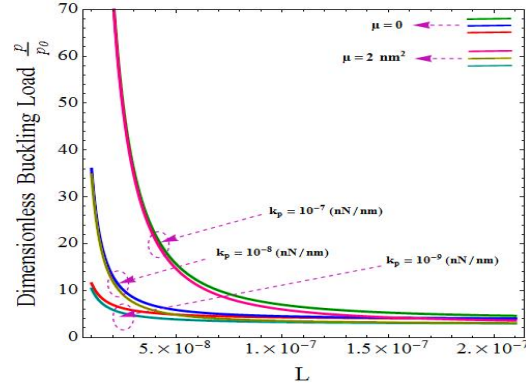
As a case study, a silver nanowire with a circular cross section will be investigated in this section. The material constants are considered as  $E=76$  GPa,  $\delta^0=0.89$  N/m and  $E^s=1.22$  N/m on the surface. Variations of buckling forces vs. the diameter of the simply supported nanowire have been represented in Fig. 2.



**Fig. 2.** The critical compressive load of a nanowire as a function of the diameter  $D$ . ( $k_w = 10^8$ ,  $n=1$ )

In Fig. 2 the influence of the elastic medium on the compressive load of buckling has been investigated.

It can be realized that attendance of the elastic medium rises the buckling forces. Alternatively the effect of surface effects on the compressive load of buckling comes to be more important as the diameter decreases. Besides generally, the influence of the nonlocal parameter  $\mu$  is to increase the buckling loads as can be comprehended from the results presented in Fig. 3.



**Fig. 3.** The critical compressive load of a nanowire as a function of the length  $L$ . ( $n=1$ ,  $k_w = 10^8$ )

In addition the buckling loads in different mode numbers and Pasternak stiffness constant ( $k_p$ ) with respect to the nanowire length as well as diameter have been stated in the Fig. 4 respectively.

Fig. 2 illustrates the normalized critical buckling load ( $P_{cr} / P_{cr0}$ ) versus nanowire diameter for different values of Pasternak spring parameters, and for local ( $\mu=0$ ) as well as nonlocal ( $\mu=2$  nm<sup>2</sup>) theories. The normalized critical load of buckling shows a different dependence on the characteristic size of the nanowire. The effect of surface effects become considerable as the diameter decreases in the range of nanometers and usually raises the critical buckling load of nanowire.

Fig. 3 displays the variation of normalized critical buckling load ( $P_{cr} / P_{cr0}$ ) with respect to the nanowire length. It can be seen that, the surface tension effects result in an effective force that tends to strengthen the nanowire. Also, as the stiffness spring  $k_w$  increases the normalized critical buckling loads increases.

Fig. 4 shows the change in the normalized critical buckling load ( $P_{cr} / P_{cr0}$ ) versus nanowire length with the nonlocal parameter  $\mu=2$  nm<sup>2</sup> and Winkler stiffness constant  $k_w = 10^8$  N/m<sup>2</sup> for selected mode numbers ( $n = 1, 2$ ) for different values of Pasternak stiffness constant. Here, it is clear that by increasing the mode number, the normalized critical buckling load decreases.

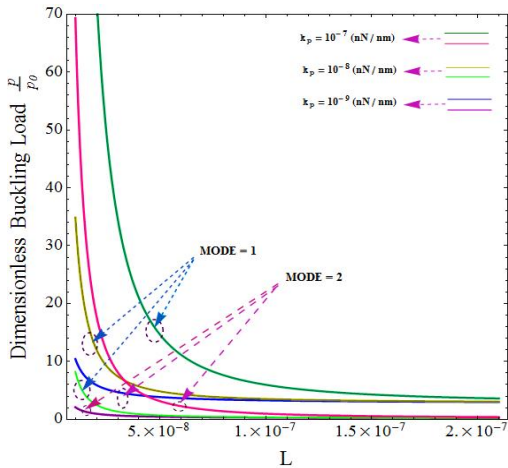


Fig. 4. The critical compressive load of a nanowire as a function of the L. ( $\mu=2 \text{ nm}^2$ ,  $k_w=10^8 \text{ N/m}^2$ )

Fig. 5 shows the variation of non-dimensional critical buckling load versus Winkler stiffness constant  $k_w$  for several selected values of the Pasternak stiffness constant and also for local and nonlocal elasticity model. It can be observed that in the case of  $k_w > 10^7 \text{ N/m}^2$  the normalized critical buckling load increase. This fact can be seen from Equation (13) mathematically. With increasing Pasternak stiffness constant, the difference between the local and nonlocal results increases due to the effect of small length scale.

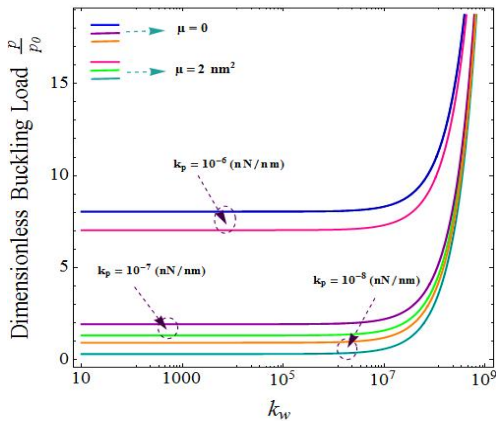


Fig.5. The critical compressive load of a nanowire as a function of the  $k_w$ . ( $n=1$ )

Fig. 6 demonstrates the normalized critical buckling load versus Pasternak stiffness constant  $k_p$  for several selected values of the Winkler stiffness constant. An

enhance in the non-dimensional critical buckling load is identified when  $k > 10^{-7} \text{ nN/nm}^2$ . This fact can be seen from Equation (13) mathematically. With growing Pasternak stiffness constant, the variance between the local and nonlocal results rises because of the influence of small length scale.

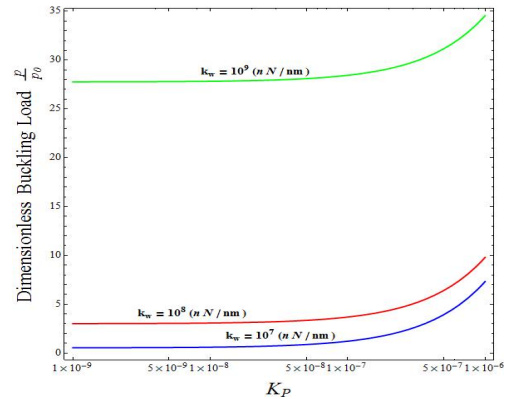


Fig. 6. The critical compressive load of a nanowire as a function of the  $k_p$ . ( $n=1$ )

## CONCLUSION

In this study, the buckling of nanowires under uniaxial load embedded in Winkler - Pasternak elastic medium has been studied based on the Euler-Bernoulli nonlocal model. It has been shown that with increasing the ratio of surface area to bulk at nano-scale, the effect of surface energy come to be noteworthy and it should be taken into consideration. The results indicate that residual surface tension, surface elasticity as well as nonlocal parameter affect the buckling behavior of nanowires.

When influence of surface is taken into consideration, the critical parameters of a nanowire correspondingly depend on its cross-sectional size. Growing the Winkler and Pasternak stiffness constants reduces the size dependency of the buckling deflection. Alternatively the impact of the Pasternak stiffness constant on the buckling load turn out to be more substantial comparing with the Winkler stiffness constant, however the elastic medium tends to raise the critical axial load.

## CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

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